M273Q Multivariable Calculus
An Old Exam 3

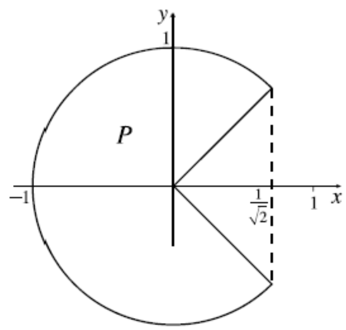
Name and section: ______________________________

Instructor's name: ______________________________

Instructions: Closed book. No calculator allowed. Double-sided exam. NO CELL PHONES. **Show all work and use correct notation to receive full credit!** Write legibly.

$dA = r\,dr\,d\theta$	$dV = r\,dz\,dr\,d\theta$	$dV = \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta$

1. Consider the region $\mathcal{P}$ given below.



(a) (1 credit ____) Describe the region P in polar coordinates using mathematically correct notation.

(b) (1 credit ____) Calculate $\displaystyle\iint_P y\,dA$

Question:	1	Total
Credit	2	2
GPA Credit Points Earned		

2. Consider the integral $\int_0^2 \int_{y^2}^4 \sqrt{1 + x^{3/2}}\, dx\, dy$.

   (a) (1 credit ____) Sketch the region of integration.

   (b) (2 credit ____) Reverse the order of integration and compute the integral.

Question:	2	Total
Credit	3	3
GPA Credit Points Earned		

3. (1 credit ____) Convert to polar coordinates to evaluate

$$\int_0^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \frac{1}{\sqrt{x^2+y^2}}\,dx\,dy.$$

Question:	3	Total
Credit	1	1
GPA Credit Points Earned		

4. Consider the triple integral

$$\int_0^1 \int_{y^3}^{\sqrt{y}} \int_0^{xy} dz\, dx\, dy$$

representing a solid $\mathcal{S}$. Let R be the projection of $\mathcal{S}$ onto the plane $z = 0$.

(a) (1 credit ____) Draw the region R.

(b) (1 credit ____) Rewrite this integral as a triple integral in the order $dz\, dy\, dx$. Do not compute the resulting integral.

Question:	4	Total
Credit	2	2
GPA Credit Points Earned		

5. (2 credit ____) A solid object occupies the region inside the cone $z = \sqrt{x^2 + y^2}$ (that is $z \geq \sqrt{x^2 + y^2}$) and between the two spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 9$.
Rewrite *BUT DO NOT EVALUATE* the triple integral in the spherical coordinate system.

$$\iiint\limits_E e^{(x^2+y^2+z^2)^{\frac{3}{2}}}\, dV$$

Question:	5	Total
Credit	2	2
GPA Credit Points Earned		

6. (2 credit ____) Use Green's Theorem to evaluate $\int_{\mathcal{C}} (2xy - y^2)\, dx + x^2\, dy$ where $\mathcal{C}$ is the boundary of the region enclosed by $y = x + 1$ and $y = x^2 + 1$, traversed in a counterclockwise manner.

Question:	6	Total
Credit	2	2
GPA Credit Points Earned		

7. Let $\mathbf{F}(x, y, z) =< 2x - y,\ z - x,\ y + 1 >$.

(a) (1 credit ____) Show that $\mathbf{F}$ is conservative. Justify your answer.

(b) (2 credit ____) Find a function f so that $\nabla f = \mathbf{F}$.

Question	Points	Score
7	3	
Total:	3	

8. For each part of problem a-d below, let $\mathcal{C}$ be the straight line segment from $(1,0,1)$ to $(0,3,6)$.

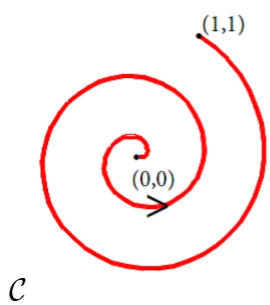
   (a) (1 credit ____) Give a parametrization for $\mathcal{C}$, the straight line segment from $(1,0,1)$ to $(0,3,6)$.

   (b) (1 credit ____) Calculate $\displaystyle\int_{\mathcal{C}} y\,dx$.

   (c) (1 credit ____) Let $\mathbf{F}$ be the vector field $\mathbf{F} =< xy, y^2, 1 >$, calculate $\displaystyle\int_{\mathcal{C}} \mathbf{F}\cdot d\mathbf{r}$.

(d) (1 credit ____) Calculate $\int_{\mathcal{C}} xy^2z\,ds$.

9. (1 credit ____) Compute $\int_{\mathcal{C}} \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F}(x,y) = \left\langle (1+xy)e^{xy}, e^y + x^2e^{xy} \right\rangle$ and $\mathcal{C}$ is as pictured. Note that if $f(x,y) = e^y + xe^{xy}$, then $\nabla f = \mathbf{F}$.



Question	Points	Score
8	4	
9	1	
Total:	5	

Page:	1	2	3	4	5	6	7	8	9	Total
Credit	2	3	1	2	2	2	3	3	2	20
GPA Credit Points Earned										