

MS Exam: ALGEBRA (January 1998)

Instructions: Attempt all problems. Show all work, but be concise.

1. Prove that all groups of order 4 are abelian. You are to use only the definition of a group. (The order of a group is the number of elements in the group.)
2. Suppose that x, y, z, t are positive integers. If $x = yz + t$, show that $\{x, z\}$ and $\{z, t\}$ have the same greatest common divisor, up to units.
3. For the rational extension fields \mathbb{Q} , $\mathbb{Q}(\sqrt{3})$, $\mathbb{Q}(i)$ and $\mathbb{Q}(-1 + \sqrt{3}i)$, draw a tower of fields diagram with the correct extension degrees labeled. Prove or disprove: There exists an algebraic number α such that $\mathbb{Q}(\sqrt{3}, i) = \mathbb{Q}(\alpha)$.