

MS Comprehensive Exam

Abstract Algebra

January 17, 2001

A finite abelian group $(\mathbf{G}, *)$ has the property that $\mathbf{g} * \mathbf{h} * \mathbf{k} = \mathbf{e}$ whenever $\mathbf{g}, \mathbf{h}, \mathbf{k}$ are distinct elements of the group \mathbf{G} . Show that \mathbf{G} has at most **3** elements.

What is the splitting field over the rationals \mathbf{Q} of the following polynomial?

$$P(x) = (x^2 + x + 1)^3 - (x^6 + x^3 + 1).$$

Recall that the splitting field is the smallest subfield of the complex numbers containing \mathbf{Q} and all of the roots of $P(x)$.

Express your answer in the form $\mathbf{Q}[\alpha]$. What is the minimal polynomial of α ?