MS Comprehensive Exam Abstract Algebra April 23, 2001

- 1. Suppose that G is a group and N is a normal subgroup of G of finite index n in G ($|G:N| = n < \infty$). Suppose that 3 divides n. Prove that there is a $g \in G$, $g \notin N$, such that $g^3 \in N$.
- 2. Prove that every ideal in the ring \mathbb{Z} of integers is a principal ideal.
- 3. Let $p(x) = x^4 6x^3 + 2x 2$. Prove that $\mathbb{Q}[x]/p(x)$ is a field.