MS Comprehensive Exam

Abstract Algebra

April 23, 2001

1. Suppose that G is a group and N is a normal subgroup of G of finite index n in G $\quad(|G:N| = n < \infty)$. Suppose that 3 divides n. Prove that there is a $g \in G, \quad g \notin N, \quad$ such that $g^3 \in N$.

2. Prove that every ideal in the ring $\mathbb{Z}$ of integers is a principal ideal.

3. Let $p(x) = x^4 - 6x^3 + 2x - 2. \quad$ Prove that $\mathbb{Q}[x]/p(x))$ is a field.