Problem 1. Let G be a multiplicative group and let n be a positive integer. Show that if G has at least one subgroup of order n, then the intersection of the family of all subgroups of G having order n is a normal subgroup of G.

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Problem 2. Suppose that G is a multiplicative group for which the quotient group $G/Z(G)$ is a cyclic group, where $Z(G)$ is the *center* of G:

$$Z(G) = \{ h \in G \mid hg = gh \text{ for every } g \in G \} .$$

Prove that G is an abelian group.

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Problem 3. Let $p(x) = x^4 + 9x^2 - 3x + 6$. Prove that the ring $\mathbb{Z}[x]/(p(x))$ is an integral domain (i. e., has no non-zero zero divisors).

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Problem 4. Let F be a field. Prove that $F[x]$ principal ideal domain (i. e., every ideal of $F[x]$ is a principal ideal).

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