

ANALYSIS: Master's Comprehensive Exam

Instructions: Attempt all of the problems, showing work. Place at most one problem solution on a side for each sheet of paper turned in. Do not submit your scratch work.

1. Define the function $f(x) = \sum_{n=0}^{\infty} \left(\frac{\sin x}{n+1} \right)^2$. For what values of $x \in \mathbf{R}$ is f continuous? Justify your answer.

2. Suppose that $f : K \rightarrow \mathbf{R}^m$ is continuous on the compact set $K \subset \mathbf{R}^n$. Define *uniform continuity*. Prove that f is uniformly continuous.

3. Suppose that $f : [0, 1] \rightarrow \mathbf{R}$ is a continuous function.

(a) Evaluate $\lim_{n \rightarrow \infty} \int_0^1 x^n f(x) dx$. Be sure to prove your answer is correct.

(b) Evaluate $\lim_{n \rightarrow \infty} n \int_0^1 x^n f(x) dx$. Be sure to prove your answer is correct.