

Dynamical Systems M.S. exam 2002

1. Find bifurcation value(s) of the parameter $r \in \mathbf{R}$ and describe the type of the bifurcation(s) for a differential equation

$$\dot{x} = x - \frac{rx}{1+x^2}.$$

Also, draw bifurcation diagram and phase portrait before, at and after the bifurcation.

2. Consider the following system of differential equations in the positive quadrant $Q := \{(x, y) : x, y \geq 0\}$

$$\dot{x} = x(1 - x - 2y) \tag{1}$$

$$\dot{y} = y(1 - y - x/2). \tag{2}$$

- a) Find and classify all the equilibrium points.
- b) Show that there is $l > 0$ such that $R := [0, l] \times [0, l]$ is positively invariant (i.e. if an initial condition is in R then so is the whole solution curve for $t \geq 0$).
- c) Show that there are no periodic orbits.
- d) Show that there is a heteroclinic orbit between the two non-zero equilibria.

3. Let $f : [0, 1] \rightarrow [0, 1]$ be as depicted below.

- a) How many distinct periodic orbits of period 3 are there?
- b) Show that there exists $x \in [0, 1]$ that is neither periodic nor eventually periodic (under f).
- c) Show that there is $x \in [0, 1]$ with a dense orbit in $[0, 1]$.