M.S. Dynamical Systems Exam 2004

(Department of Mathematical Sciences, M.S.U.)

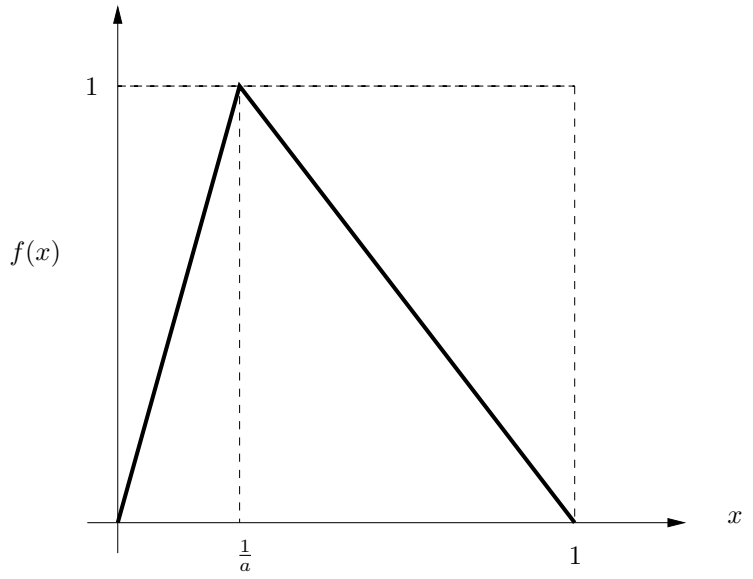
Instructions: Attempt all questions. Show all work.

1. Consider the following system of differential equations in $\mathbb{R}^2$ in polar coordinates:

$$\begin{aligned} \dot{r} &= r(\mu - r^2) \\ \dot{\theta} &= \sin^2\theta - \mu + 2 \end{aligned}$$

where $\mu \geq 0$ is a bifurcation parameter. For all $\mu \geq 0$ determine the existence and stability of all periodic orbits and fixed points.

2. Let $f : [0,1] \to [0,1]$ be as depicted below.



Using the figure above draw a qualitatively accurate figure for the second iterate map $f^2(x)$ for $a > 1$. On your figure label the location of all (minimal) period 2 orbits and deduce their stability.

3. Consider the following differential equations in $\mathbb{R}^2$:

$$\begin{aligned} \dot{x} &= y + \lambda x\left(\frac{1}{2} - r\right) \\ \dot{y} &= -x + \lambda y(1 - r) \end{aligned}$$

where $r^2 = x^2 + y^2$. Determine $\dot{r}$ in polar coordinates and then prove the system above has at least one periodic orbit for $\lambda > 0$.