M.S. Dynamical Systems Exam 2005

(Department of Mathematical Sciences, M.S.U.)

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Instructions: Attempt all questions. Show all work.

1. Consider the map $f : \mathbb{R} \to \mathbb{R}$ defined by

$$x \to f(x) = \frac{x}{\mu + x}$$

where μ is a real parameter.

a) Find all the fixed points of this map and determine the μ for which they are stable.

b) Does this map have a minimal period 2 orbit for any μ? If so, what μ?

2. In polar coordinates (r, θ) planar flow is described by:

$$\frac{dr}{dt} = r(\mu + r^2 - r^4) \tag{1}$$

$$\frac{d\theta}{dt} = 1 + r^2 \sin^2 \theta \tag{2}$$

where μ is a real parameter.

a) Draw a bifurcation diagram for the system (1)-(2) in the (μ, r)-plane labelling the stability of all periodic orbits and fixed points (Use a solid line for stable and a dashed line for unstable).

b) Does the system have a Hopf bifurcation? Explain.

c) Draw a qualitatively accurate phase portrait in the (x, y)-plane for $\mu = -\frac{1}{8}$.

3. The following questions apply to the planar system:

$$\frac{dx}{dt} = x(4 - y - x^2) \tag{3}$$

$$\frac{dy}{dt} = y(1 - x) \tag{4}$$

a) Determine the coordinates and stability of all fixed points. Label the location of these fixed points in the xy-plane along with the x and y nullclines.

b) Does this system have any periodic orbits? Explain.