MS Exam: LINEAR ALGEBRA JAN. 1998

Instructions: Attempt all problems, showing work. Note: all vector spaces are finite dimensional.

1. Recall that trace(T), for a linear transformation $T : \mathbf{V} \to \mathbf{V}$, is defined to be the trace of the matrix of T in an arbitrary basis. *If you wish to use another definition of trace, be sure to prove your definition is equivalent.* Now suppose that $(\mathbf{V}, \langle\cdot,\cdot\rangle)$ is a complex inner product space and define the linear transformation

$$T\mathbf{v} := \langle \mathbf{v}, \mathbf{u} \rangle \mathbf{w}$$

for fixed vectors $\mathbf{u}, \mathbf{w} \in \mathbf{V}$. Calculate trace($T$).

2. Let $(\mathbf{V}, \langle\cdot,\cdot\rangle)$ denote a complex inner product space. Suppose that T commutes with its adjoint T^*; i.e., T is *normal*. Show that if $T^2 v = 0$ for some $v \in \mathbf{V}$, then $Tv = 0$.

3. Determine the Jordan form matrix J corresponding to the matrix

$$A = \begin{pmatrix} -4 & -8 & 4 \\ -1 & 0 & 1 \\ -8 & -12 & 8 \end{pmatrix}$$

Explicitly determine the matrix P such that $P^{-1}AP = J$.

Hint: The matrix is not invertible.