MS Exam: LINEAR ALGEBRA March 98

Instructions: Attempt all problems, showing work. Note: all vector spaces are finite dimensional over $\mathbf{C}$.

1. Suppose a linear transformation $T : \mathbf{V} \to \mathbf{V}$ is such that $T^2 = 0$. Recall that $rank(T)$ is the dimension of the range of T. If $Dim(V) = n$, show $rank(T)$ is at most $\frac{n}{2}$.

2. Let $(\mathbf{V}, \langle \cdot, \cdot \rangle)$ denote a complex inner product space. Recall that a transformation $T : V \to V$ is *self-adjoint* iff $T = T^*$, where T^* denotes the adjoint.
   (a) Prove that every self-adjoint operator T on V has a cube root S (so that $S^3 = T$)
   (b) Is S unique? Prove or disprove.

3. Determine the Jordan form matrix J corresponding to the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Explicitly determine the matrix P such that $P^{-1}AP = J$.