MS Exam: LINEAR ALGEBRA JAN 1999

Instructions: Attempt all problems, showing work. Note: all vector spaces are finite dimensional.

1. For a linear transformation $T : \mathbf{V} \to \mathbf{V}$, where $\mathbf{V}$ is a complex vector space of dimension n, $0 < n < \infty$, prove (using only the definitions) that if T has n *distinct* eigenvalues, then T is diagonalizable. Give a counterexample for $n = 2$ of the *converse* statement.

2. Let $(\mathbf{V}, \langle \cdot, \cdot \rangle)$ denote a complex inner product space. Suppose that T commutes with its adjoint T^* and $T^2 = T$. Show that T is an *orthogonal projection*. Thus, you need to show that there are subspaces $\mathbf{U}, \mathbf{W}$ of $\mathbf{V}$ such that

$\mathbf{V} = \mathbf{U} + \mathbf{W}$,
$\mathbf{U} \perp \mathbf{W}$,
range$(T) = \mathbf{W}$ and
nullspace$(T) = \mathbf{U}$.

3. For parts (a) and (c) you may answer without proof.
(a) Find the jordan form of the following matrix (the field is $\mathbf{C}$).

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$$

(b) Find the jordan form of the following matrix (the field is $\mathbf{C}$).

$$B = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$

(c) Find all possible jordan forms for a (3×3)-nilpotent matrix N. Recall that N is nilpotent if $N^k = 0$ for some positive integer k.