LINEAR ALGEBRA: Master's Comperehensive Exam JAN 2005

Instructions: Attempt all of the problems, showing work. Place at most one problem on a side for each sheet of paper turned in. Do not submit scratch work. Note: all vector spaces are *finite dimensional*.

1. Let $\mathcal{M}_3(\mathbf{R})$ denote the vector space of all 3×3 (real) matrices. Let L denote the subspace of all such matrices having the property that the sum of the entries in every row is zero. Determine the dimension of L, and prove your answer is correct. (It is not necessary to show that L is a subspace.)

2. Let $(\mathbf{V}, \langle \cdot, \cdot \rangle)$ denote a complex inner product space. Suppose that $T : \mathbf{V} \to \mathbf{V}$ is a linear transformation such that its adjoint T^* satisfies the equation $T^* = -T$.

(a) Show that the transformation $T - I$ is invertible, where I denotes the identity transformation.

(b) Show that the transformation $S = (T + I)(T - I)^{-1}$ has the property that $||Sv|| = ||v||$ for all $v \in \mathbf{V}$.

3. Determine the Jordan form matrix J corresponding to the matrix

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 0 & 2 \\ 0 & -1 & 3 \end{pmatrix}$$

Explicitly, find a matrix P such that $P^{-1}AP = J$.