

LINEAR ALGEBRA: Master's Comprehensive Exam JAN 2005

Instructions: Attempt all of the problems, showing work. Place at most one problem on a side for each sheet of paper turned in. Do not submit scratch work. Note: all vector spaces are *finite dimensional*.

1. Let $\mathcal{M}_3(\mathbf{R})$ denote the vector space of all 3×3 (real) matrices. Let L denote the subspace of all such matrices having the property that the sum of the entries in every row is zero. Determine the dimension of L , and prove your answer is correct. (It is not necessary to show that L is a subspace.)

2. Let $(\mathbf{V}, \langle \cdot, \cdot \rangle)$ denote a complex inner product space. Suppose that $T : \mathbf{V} \rightarrow \mathbf{V}$ is a linear transformation such that its adjoint T^* satisfies the equation $T^* = -T$.

(a) Show that the transformation $T - I$ is invertible, where I denotes the identity transformation.

(b) Show that the transformation $S = (T + I)(T - I)^{-1}$ has the property that $\|Sv\| = \|v\|$ for all $v \in \mathbf{V}$.

3. Determine the Jordan form matrix J corresponding to the matrix

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 0 & 2 \\ 0 & -1 & 3 \end{pmatrix}$$

Explicitly, find a matrix P such that $P^{-1}AP = J$.