

Linear Algebra Masters Comp

March 23, 2005

1. Suppose that T is a linear operator on the finite dimensional vector space V . Prove that the following are equivalent:
 - (a) $V = \text{null}T \oplus \text{range}T$
 - (b) $\text{null}T \cap \text{range}T = \{0\}$
 - (c) $\text{null}T = \text{null}T^2$.
2. Suppose that $\| \cdot \|$ is a norm on the real finite dimensional vector space V . What (if any) conditions are required to insure that there is an inner product $\langle \cdot, \cdot \rangle$ on V so that $\|v\|^2 = \langle v, v \rangle$ for all $v \in V$? Explain.
3. Let $P_n(\mathbb{C})$ be the complex vector space of polynomials of degree $\leq n$ with complex coefficients. Let T be the operator on $P_n(\mathbb{C})$ defined by $T(p(z)) = p'(z)$ (the derivative operator). Find:
 - (a) the characteristic polynomial of T
 - (b) the minimal polynomial of T
 - (c) a Jordan basis for T
 - (d) the Jordan form of T .