Linear Algebra Masters Comp

March 23, 2005

1. Suppose that T is a linear operator on the finite dimensional vector space V. Prove that the following are equivalent:

   (a) $V = nullT \oplus rangeT$

   (b) $nullT \cap rangeT = \{0\}$

   (c) $nullT = nullT^2$.

2. Suppose that $\| \ \|$ is a norm on the real finite dimensional vector space V. What (if any) conditions are required to insure that there is an inner product $< \ , \ >$ on V so that $\|v\|^2 = < v, v >$ for all $v \in V$? Explain.

3. Let $P_n(\mathbb{C})$ be the complex vector space of polynomials of degree $\leq n$ with complex coefficients. Let T be the operator on $P_n(\mathbb{C})$ defined by $T\,(p(z)) = p'(z)$ (the derivative operator). Find:

   (a) the characteristic polynomial of T

   (b) the minimal polynomial of T

   (c) a Jordan basis for T

   (d) the Jordan form of T.