M.S. Linear Algebra Exam 2006

(Department of Mathematical Sciences, M.S.U.)

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Instructions: Attempt all questions. Show all work.

1. Let $\mathcal{M}$ be the vector space of real $n \times n$ matrices and let V be the subspce

$$V = \{X \in \mathcal{M} : \ trace\ X = 0\}$$

What is $dim(V)$?

2. Let V be a real inner product space and suppose that $v, w \in V$ are such that $< v, w > \neq 0$. Find a formula for the scalar t that minimizes $\| v - tw \|$.

3. Let V be a finite dimensional inner product space and let T be a self-adjoint linear operator on V. Given a positive integer k, prove that there is a linear operator S on V with $S^{2k+1} = T$. Is such an S necessarily self-adjoint?

4. Let L be a linear operator on $\mathbb{R}^5$ with the properties: there are vectors $u, v, w \in \mathbb{R}^5$ with v and w linearly independent, so that

$$(L - 2I)^2 u \neq 0 \quad , \quad (L - 2I)^3 u = 0 \quad , \quad Lv = 0 \quad , \quad Lw = 0 \ .$$

Find:

a) the characteristic polynomial of L

b) the minimal polynomial of L

c) the Jordan form of L

d) a Jordan basis for L.