

*M.S. Linear Algebra Exam 2006*  
(DEPARTMENT OF MATHEMATICAL SCIENCES, M.S.U.)  
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**Instructions:** Attempt all questions. Show all work.

1. Let  $\mathcal{M}$  be the vector space of real  $n \times n$  matrices and let  $V$  be the subspace

$$V = \{X \in \mathcal{M} : \text{trace } X = 0\}$$

What is  $\dim(V)$ ?

2. Let  $V$  be a real inner product space and suppose that  $v, w \in V$  are such that  $\langle v, w \rangle \neq 0$ . Find a formula for the scalar  $t$  that minimizes  $\|v - tw\|$ .

3. Let  $V$  be a finite dimensional inner product space and let  $T$  be a self-adjoint linear operator on  $V$ . Given a positive integer  $k$ , prove that there is a linear operator  $S$  on  $V$  with  $S^{2k+1} = T$ . Is such an  $S$  necessarily self-adjoint?

4. Let  $L$  be a linear operator on  $\mathbb{R}^5$  with the properties: there are vectors  $u, v, w \in \mathbb{R}^5$  with  $v$  and  $w$  linearly independent, so that

$$(L - 2I)^2 u \neq 0 \quad , \quad (L - 2I)^3 u = 0 \quad , \quad Lv = 0 \quad , \quad Lw = 0 .$$

Find:

- a) the characteristic polynomial of  $L$
- b) the minimal polynomial of  $L$
- c) the Jordan form of  $L$
- d) a Jordan basis for  $L$ .