Linear Algebra Masters Exam

April 4, 2006

1. Let $V = L(\mathbb{R}^{2006}, \mathbb{R}^2)$ be the vector space of linear transformations from $\mathbb{R}^{2006}$ to $\mathbb{R}^2$ and let $v_0 \in \mathbb{R}^{2006}$ be a nonzero vector. Define a linear transformation $E : V \to \mathbb{R}^2$ by $E(T) = T(v_0)$. Find $\dim(\text{null } E)$.

2. Suppose that $T \in L(V)$ is a linear operator on the n-dimensional vector space V. Let W be the subspace of $L(V)$ spanned by $\{I, T, T^2, \cdots\}$. Prove that $\dim(W) \leq n$.

3. Suppose that V is a complex vector space of dimension 5, $T \in L(V)$ is nilpotent, and $\dim(\text{null } T) = 2$. Find all possible minimal polynomials of T and the corresponding Jordan cannonical forms of T.

4. Suppose that V is a real finite dimensional inner-product space and that W is a subspace of V. Let $T : V \to V$ be the orthogonal projection onto W. Prove that T is self-adjoint.