

# Linear Algebra Masters Exam

April 4, 2006

1. Let  $V = L(\mathbb{R}^{2006}, \mathbb{R}^2)$  be the vector space of linear transformations from  $\mathbb{R}^{2006}$  to  $\mathbb{R}^2$  and let  $v_0 \in \mathbb{R}^{2006}$  be a nonzero vector. Define a linear transformation  $E : V \rightarrow \mathbb{R}^2$  by  $E(T) = T(v_0)$ . Find  $\dim(\text{null } E)$ .
2. Suppose that  $T \in L(V)$  is a linear operator on the  $n$ -dimensional vector space  $V$ . Let  $W$  be the subspace of  $L(V)$  spanned by  $\{I, T, T^2, \dots\}$ . Prove that  $\dim(W) \leq n$ .
3. Suppose that  $V$  is a complex vector space of dimension 5,  $T \in L(V)$  is nilpotent, and  $\dim(\text{null } T) = 2$ . Find all possible minimal polynomials of  $T$  and the corresponding Jordan canonical forms of  $T$ .
4. Suppose that  $V$  is a real finite dimensional inner-product space and that  $W$  is a subspace of  $V$ . Let  $T : V \rightarrow V$  be the orthogonal projection onto  $W$ . Prove that  $T$  is self-adjoint.