

Linear Algebra M.S. Comprehensive Examination

January 2007

Name:

1. An $n \times n$ matrix P is called a *permutation matrix*, if its rows are the same as the rows of the identity matrix, but in arbitrary order.

How many 5×5 permutation matrices are there? Are they linearly independent? Do they span the space of all 5×5 matrices?

(You do not have to write them all down.)

2. Suppose P is an orthogonal projection onto a subspace S and Q is an orthogonal projection onto the orthogonal complement S^\perp .

a. Describe the operators $P + Q$ and PQ .

b. Show that $P - Q$ is its own inverse.

3. Show that the inverse of an invertible Hermitian operator is also Hermitian.

4. True or false. Correct guesses do not count - you must either give a counterexample or a proof.

a. An invertible matrix cannot be similar to a singular matrix.

b. A symmetric matrix cannot be similar to a non-symmetric matrix.

c. A matrix B cannot be similar to $-B$ unless $B = 0$.