

Masters Examination:
Linear Algebra
January, 2009.

Instructions: Answer all of the following questions

1. Find all solutions of the system of equations

$$\begin{aligned}x + y + z &= 1 \\3x + y - z &= 0 \\2x + y &= 1.\end{aligned}$$

2. Prove that

$$e^A + I = 0$$

where

$$A = \begin{pmatrix} 0 & \pi \\ -\pi & 0 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad 0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

3. Suppose V is a finite-dimensional inner product space and that $\mathbf{w} \in V$ with $\mathbf{w} \neq \mathbf{0}$. Define T by

$$T\mathbf{u} = \langle \mathbf{u}, \mathbf{w} \rangle \frac{\mathbf{w}}{\|\mathbf{w}\|^2}.$$

A. Show that T is linear.

B. Calculate the trace of T .

C. Find the minimal and characteristic polynomials for T .

4. Suppose that $A\mathbf{x} = \mathbf{b}$, where A is a 2×2 matrix and \mathbf{b} is a 2-vector, has more than one solution. Describe a method to find the smallest solution, i.e., to find the minimum of $\|\mathbf{x}\|$ over all solutions to $A\mathbf{x} = \mathbf{b}$.