

LINEAR ALGEBRA MASTER'S EXAM

January 2011

Instructions: Solve 4 of the following 5 questions. Show all work. Carefully read and follow the directions. Clearly label your work and attach it to this sheet.

1. Suppose that z_1, z_2, \dots, z_{m+1} are distinct elements of the field \mathbf{F} and that $w_1, w_2, \dots, w_{m+1} \in \mathbf{F}$. Prove that there exists a unique polynomial $p \in \mathcal{P}_m(\mathbf{F})$

$$p(z_j) = w_j,$$

for $j = 1, 2, \dots, m + 1$.

2. Suppose that V is a real finite dimensional inner-product space and that W is a subspace of V . Let $T : V \rightarrow V$ be the orthogonal projection onto W . Prove that T is self-adjoint.
3. Let $V = P_3(\mathbb{R})$, and define $T \in \mathcal{L}(V)$ by

$$T(a + bx + cx^2 + dx^3) = -d + (-c + d)x + (a + b - 2c)x^2 + (-b + c - 2d)x^3$$

Let $B = (1 - x + x^3, 1 + x^2, 1, x + x^2)$ denote a basis of V .

- (a) Compute $\mathcal{M}(T)$, the matrix of T with respect to the basis B .
 - (b) Determine whether or not B is a basis consisting of eigenvectors of T .
4. Let $V = P_3(\mathbb{R})$ and $Tp = p(x) + p(2)x$. Find the eigenvalues and corresponding eigenvectors of T .
 5. Find a Singular Value Decomposition for

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \end{bmatrix}$$