LINEAR ALGEBRA MASTER'S EXAM

January 2011

Instructions: Solve 4 of the following 5 questions. Show all work. Carefully read and follow the directions. Clearly label your work and attach it to this sheet.

1. Suppose that $z_1, z_2, \ldots z_{m+1}$ are distinct elements of the field $\mathbf{F}$ and that $w_1, w_2, \ldots w_{m+1} \in \mathbf{F}$. Prove that there exists a unique polynomial $p \in \mathcal{P}_m(\mathbf{F})$
$$p(z_j) = w_j,$$
for $j = 1, 2, \ldots, m+1$.

2. Suppose that V is a real finite dimensional inner-product space and that W is a subspace of V. Let $T : V \to V$ be the orthogonal projection onto W. Prove that T is self-adjoint.

3. Let $V = P_3(\mathbb{R})$, and define $T \in \mathcal{L}(\mathcal{V})$ by
$$T(a+bx+cx^2+dx^3) = -d+(-c+d)x+(a+b-2c)x^2+(-b+c-2d)x^3$$
Let $B = (1-x+x^3, 1+x^2, 1, x+x^2)$ denote a basis of V.
   (a) Compute $\mathcal{M}(\mathcal{T})$, the matrix of T with respect to the basis B.
   (b) Determine whether or not B is a basis consisting of eigenvectors of T.

4. Let $V = P_3(\mathbb{R})$ and $Tp = p(x) + p(2)x$. Find the eigenvalues and corresponding eigenvectors of T.

5. Find a Singular Value Decomposition for
$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \end{bmatrix}$$