STAT 501 Comprehensive Exam - January 4, 2012

Note: Start each problem on a new page. You may use the equation sheet from class with Chebychev's and Jensen's Inequalities added to it.

1. Approximately 1/125 of all births are fraternal twins and 1/300 are identical twins. Elvis Presly had a twin brother (who died at birth). What is the probability that Elvis was an identical twin? You may approximate the probability of a boy or girl birth as 1/2.

2. Let $P(A) = P(B) = 1/3$ and $P(A \cap B) = 1/10$. Find

   (a) $P(A^c \cup B^c)$.

   (b) $P(B \cap A^c)$.

3. A certain river floods every year. Suppose that the low-water mark is set at 1, and the high water mark Y has cdf

$$F_Y(y) = P(Y \leq y) = \begin{cases} 1 - 1/y^2, & 1 \leq y < \infty \\ 0 & elsewhere \end{cases}$$

   (a) Verify that F_Y is a cdf.

   (b) Find the pdf of Y.

   (c) The low water mark is reset at 0 and a unit of measurement equal to 1/10 of that given previously is used. Let Z denote the new high water mark. Find Z in terms of Y and find $F_Z(z)$.

4. Let $X|N = n$ be $Gam(n, \beta)$ and $N \sim Geom(p)$.

   (a) Find the marginal distribution of X.

   (b) Find the $E(X)$ and $Var(X)$.

5. Let $X \sim Gam(r, 1)$ and $Y \sim Gam(s, 1)$ with X and Y independent. Let $U = X + Y$ and $V = X/U$.

   (a) Find the joint distribution of U and V. Are U and V independent? Justify your answer.

   (b) Find the marginal distribution of V.

6. For random variables X and Y, function $g(x, y)$ and constants a and b show that if $a \leq g(x, y) \leq b$ then $a \leq E(g(X, Y)) \leq b$. You may assume that X and Y are continuous random variables.

7. Suppose $X_1, \cdots, X_n \overset{ind}{\sim} N(\mu, \sigma^2)$. We know that $(n-1)S^2/\sigma^2 \sim \chi^2_{n-1}$. Use this fact to find $E(S)$.

8. Let $X_1, \cdots, X_n \overset{ind}{\sim} Geom(p)$. Find the distribution of $Y = \sum X_i$.

9. Let X, Y, and Z be three random variables with common variance σ^2 and pairwise correlations $\rho_{XY} = 0.3$, $\rho_{XZ} = 0.5$ and $\rho_{YZ} = 0.2$. Find $\text{Cor}(X + Y, Y + Z)$.