M.S. Numerical Exam 2001

(Department of Mathematical Sciences, M.S.U.)

Instructions: Attempt all questions. Show all work.

1. You wish to solve the linear system $Ax = b$ where A is given by:

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 3 \end{bmatrix}$$

   Determine whether or not the Gauss-Seidel iteration scheme is guaranteed to converge for all initial guesses $x_0 \in \mathbb{R}^2$.

2. Use one iteration of Newton's method for systems to determine an approximate value for the root of the equations

$$\begin{aligned} 3x_1^2 - x_2^2 - 2 &= 0, \qquad (1) \\ x_1 x_2 + x_2 - 2 &= 0. \qquad (2) \end{aligned}$$

   Use the initial guess $\mathbf{x}^0 = (x_1, x_2) = (1, 0)$.

3. Let $Q(f)$ be the 2-point quadrature approximation

$$Q(f) = af\left(-\frac{1}{3}\right) + bf(1)$$

   of $I(f) \equiv \int_{-1}^{1} f(x)dx$. Find the values of a and b which maximize the degree of the quadrature approximation.

4. Let $y(t)$ be the solution of the scalar initial value problem

$$y'(t) = f(y(t), t)\ , \quad y(0) = y_0.$$

   Further define the RK2 method

$$y_{k+1} = y_k + hf\left(y_k + \frac{1}{2}hf_k, t_k + \frac{1}{2}h\right) \quad , \quad k = 0, 1, 2, \ldots \qquad (3)$$

   where $f_k \equiv f(y_k, t_k), t_k = kh$ and $h > 0$ is the step size for the method. If the method is applied to solving $y'(t) = -10y(t)$, for what h will the method be absolutely stable?