M.S. Numerical Exam 2002

(Department of Mathematical Sciences, M.S.U.)

Instructions: Attempt all questions. Show all work.

1. Consider the system

$$x^2 + \frac{1}{3}y^3 = 9, \quad (1)$$
$$y^2 - x = 9. \quad (2)$$

Let $\mathbf{x}^{(n)} = (x_n, y_n)^T, n \geq 0$, be the n-th iterate of a Newton's Method approximation of a root of (1)-(2). Compute $\mathbf{x}^{(1)}$ for the initial guess $\mathbf{x}^{(0)} = (0, 1)^T$.

2. Let $U \in \mathbb{R}^{n \times n}$ be a nonsingular upper triangular matrix and e_j be the j-th column of the $n \times n$ identity matrix. Solutions x of $Ux = e_j$ have the form $x = (x_1, x_2, \ldots, x_j, 0, \ldots, 0)^T, 1 \leq j \leq n$. Given this special solution structure, how many flops are required to solve

$$Ux = e_j + e_{j-1}$$

using backward substitution? Assume that all upper off diagonal elements of the matrix U are nonzero.

3. Let $Q(f)$ be the 2-point quadrature approximation

$$Q(f) = f\left(-\frac{\sqrt{3}}{3}\right) + f\left(\frac{\sqrt{3}}{3}\right)$$

of the integral $I(f) \equiv \int_{-1}^{1} f(x)dx$.

a. Define the term **degree of accuracy** for the quadrature rule Q.

b. Through explicit calculation, compute the degree of accuracy for the quadrature rule Q.

4. Consider the scalar initial value problem

$$\frac{dy}{dt} = f(y, t)$$
$$y(0) = 0$$

The Runge-Kutta method of order 2 known as Heun's Method is defined by the following

$$w_{i+1} = w_i + \frac{h}{2}\left[f(w_i, t_i) + f(w_i + hf(w_i, t_i), t_{i+1})\right], \quad (3)$$

for $i = 0, 1, 2, \ldots$ where $t_i = ih$, $h > 0$ is the step size for the method and w_i is the approximation for $y(t_i)$. Show that if $f(0, t) = 0$ the global error at t_i for the method is $|y(t_i)|$.