M. Sc. Comprehensive Exam - Topology

January 2014

(1) Consider the excluded point topology $\mathcal{T} = \{R\} \cup \{U \subset R | 0 \notin U\}$ on the set of real numbers R.

(a): Describe the closure $cl(A)$ of any subset $A \subset R$ in this topology.

(b): Describe the set of limit points A' of any subset $A \subset R$ in this topology.

(c): Determine whether or not this topological space is connected and prove your assertion.

(2) Let E be a subset of a metric space X. If E is both open and closed, then we can conclude that

(a): E is the empty set.

(b): X is the empty set.

(c): X is disconnected.

(d): E is disconnected.

(e): E is either the empty set, or the whole space X.

In each case, justify your answer by a counterexample, or a proof.

(3) **(a):** Let X be a Hausdorff space and A a compact subset of X. Prove that A is closed.

(b): Let $f : X \to Y$ be a continuous map from a compact space X to a Hausdorff space Y . Prove that f is a closed map.

(4) Let $p : E \to B$ be a covering map with connected base space B, and let x and y be two points in B . Show that the sets $p^{-1}(x)$ and $p^{-1}(y)$ have the same cardinality.