

**Topology M.S. exam.**

1. Prove or disprove: If  $U$  is an open set then

$$U = \text{int}(\bar{U}).$$

Here  $\text{int}(U)$  denotes the interior and  $\bar{U}$  denotes the closure of the set  $U$ .

2. Show that every compact Hausdorff space is normal.
3. A relation  $\sim$  on  $X$  is an *equivalence relation* if
  1.  $x \sim x$  for all  $x \in X$ ;
  2. if  $x \sim y$  then  $y \sim x$  for all  $x, y \in X$ ;
  3. if  $x \sim y$  and  $y \sim z$  then  $x \sim z$  for all  $x, y, z \in X$ .

Let  $X$  be a connected space. Suppose that  $\sim$  is an equivalence relation on  $X$  with the property that, for each  $x \in X$  there is an open neighborhood  $U_x$  of  $x$  such that

$$x \sim y \quad \text{for all} \quad y \in U_x.$$

Prove that  $x \sim y$  for all  $x, y \in X$ .