

Topology M.S. exam 2002

1. The symmetric difference of sets is determined by

$$C\Delta D := C \cup D - (C \cap D).$$

Let A and B be subsets of the topological space X . Prove or give a counterexample

i.) $\overline{A\Delta B} \subset \bar{A}\Delta\bar{B}$

ii.) $\overline{A\Delta B} \supset \bar{A}\Delta\bar{B}$

2. Let $f : S^1 \rightarrow S^1$ be a continuous map (S^1 denotes the unit circle in the plane, centered at the origin, with subspace topology.) Assume that f is not a surjective map. Show that there exists a point $y \in S^1$ such that

$$f(y) = f(-y).$$

3. A topological space X is *locally connected* if for every $x \in X$ and every neighborhood U of x there is a connected neighborhood V of x with $V \subset U$.

Suppose that the topological space X is locally connected and compact. Prove that X has only finitely many connected components (maximal connected subsets).