

**MS Comprehensive Exam**  
**Topology**  
**July 2002**

1. Let  $X$  and  $Y$  be nonempty spaces. Prove that if  $X$  and  $Y$  are connected, then so is  $X \times Y$ . Is the converse true?
2. Let  $X$  be a compact space, let  $Y$  be a Hausdorff space and suppose that  $f : X \rightarrow Y$  is a continuous surjection. Is the image of every closed set closed? Is the image of every open set open?
3. Let  $(X, d)$  be a compact metric space. Prove that every compact subset of  $X$  is bounded. Is it true that every closed and bounded subset of  $X$  is compact?