MS Comprehensive Exam
Topology (vers. 1)
January 13, 2004

Problem 1. If $A \subset X$ and $B \subset Y$, show that

$$\mathrm{Bd}(A \times B) = (\mathrm{Bd}(A) \times B) \cup (A \times \mathrm{Bd}(B))$$

Problem 2. Show that if $f, g : X \to Y$ are continuous, where Y is a Hausdorff space, then the set

$$F = \{x \in X \mid \ f(x) = g(x)\}$$

is a closed subset of X.

Problem 3. Show that if $f, g : X \to Y \times Z$ are continuous, then the space

$$W = \{(x, f(x), g(x)) \mid x \in X\} \subset X \times Y \times Z$$

is homeomorphic to X.

Problem 4. Suppose that (X, d) is a compact metrizable space which has the following property: for any points x and y of X, and for any $\epsilon > 0$, there exists a finite sequence $\{x = x_0, x_1, x_2, \ldots, x_n = y\}$ of points of X such that $d(x_{i-1}, x_i) < \epsilon$ for $i \in \{1, 2, \ldots, n\}$. Prove that X is connected.

Problem 5. Let X and Y be the two subspaces of the plane $\mathbb{R}^2$ defined as follows: $(a, b) \in X$ if and only if a and b are both irrational; $(a, b) \in Y$ if and only if a is irrational or b is irrational, or both. Prove that one of the spaces X or Y is connected and the other is not.