

Topology Comprehensive Exam

April 15, 2004

1. Let X be a topological space, let $X \times X$ have the product topology, and let $\Delta = \{(x, x) : x \in X\} \subset X \times X$. Prove that X is Hausdorff if and only if Δ is closed in $X \times X$.
2.
 - a) Suppose the C is a closed subset of the compact space X . Prove that C is compact.
 - b) Suppose that C is a compact subset of the topological space X . Give an example to show that C need not be closed. What additional assumption will guarantee that C is closed?
3. Let $S^1 \subset \mathbb{R}^2$ be the unit circle, $S^1 = \{(x, y) : x^2 + y^2 = 1\}$, and let S^1 have the usual topology. Prove that if $f : S^1 \rightarrow \mathbb{R}$ is continuous then $f(S^1)$ is a closed and bounded interval.