

M. Sc. Comprehensive Exam - Topology

January 2012

In the following, $cl(S)$ denotes the closure of the set S .

- (1) a) Assume X and Y are topological spaces, and that $f : X \rightarrow Y$ is a continuous function. Let $A \subset B$. Prove that $f(cl(A)) \subset cl(f(A))$.
- (2) If f and f' are path-homotopic paths from x_0 to x_1 , and g and g' are path-homotopic paths from x_1 to x_2 , then prove that $f * g$ is path-homotopic to $f' * g'$.
- (3) a) Prove that the continuous image of a path connected space is path connected.
b) Prove that $\mathbb{R}^n \setminus \{0\}$ is path connected for every $n > 1$.
c) Use a) and b) to prove that S^{n-1} is path connected.
d) Prove that a finite product of connected spaces is connected.
- (4) Let A and B be disjoint compact subsets of the Hausdorff space X . Prove there exist disjoint open neighborhoods U and V of A and B , respectively.