Ph.D. Comprehensive Examination: Complex Analysis
August 21, 2006.

Instructions: Attempt all questions.

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1. Using the principal branch definition for z^i determine the set of all $z \in \mathbb{C}$ for which $(z^i)^2 = (z^2)^i$.

2. Suppose $f(z)$ is analytic on $|z| < 127$, has zero as a root and $Im(f(z)) = y(4x-1), z = x + iy$. Determine a formula for $f(z)$.

3. The function

$$f(z) = \frac{1}{z^2 Log(1+4z)}$$

has a pole of order 3 at $z = 0$ (here $Log(z)$ is principal branch).

a) Determine the first three terms a Laurent expansion for $f(z)$ convergent on an annular region centered at $z = 0$.

b) Compute the integral

$$I = \int_{|z|=0.00001} f(z)\, dz$$

where the orientation of the circle is counterclockwise.

4. Using an appropriate branch for $z \mapsto \sqrt{z}$, evaluate the following indefinite integral:

$$I = \int_1^\infty \frac{\sqrt{x-1}}{x^2}\, dx$$

5. Let

$$f(z) = \frac{i-z}{i+z}$$

and define

$$S(z) = \sum_{n=0}^{\infty} f(z)^n$$

a) By using the binomial theorem and summing the series $S(z)$, find a meromorphic function $g(z)$ defined for all $z \neq 0$ that is equal to $S(z)$ wherever it converges.

b) Using the fact that $f(z)$ is a linear fractional transformation, determine the set of z for which $S(z)$ converges.