Ph.D. Comprehensive Examination:
Fluid Dynamics

August 22, 2007.

Instructions: Answer all of the following questions

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1. Non-dimensionalize the time-dependent Stokes equations including a time-dependent forcing function. Under what circumstances can the time-derivative term be neglected?

2. Consider incompressible flow in a bounded region. What equation and boundary conditions determine the pressure p?

3. Solve the Navier-Stokes equations to determine the flow between 2 parallel, infinite, flat plates spaced a distance h apart, where the top plate moves with velocity U_1, the bottom plate moves with velocity U_2, and a constant (in space and time) pressure gradient $dp/dx = C$ is imposed.

4. Show that there is at most one ideal flow ($\nabla \cdot \mathbf{u} = 0$, $\nabla \times \mathbf{u} = \mathbf{0}$) in a bounded, simply connected region V if the normal flow $\mathbf{u} \cdot \mathbf{n}$ is prescribed on the boundary of V. (You can assume that boundary of V is smooth.)

(b) Show that the flow of part (a) (assuming it exists) has less kinetic energy than any other incompressible flow on V satisfying the same boundary conditions. (Hint: consider the kinetic energy of the difference $\mathbf{u} - \mathbf{v}$ of velocities.)