Mathematical Imaging & Data Science

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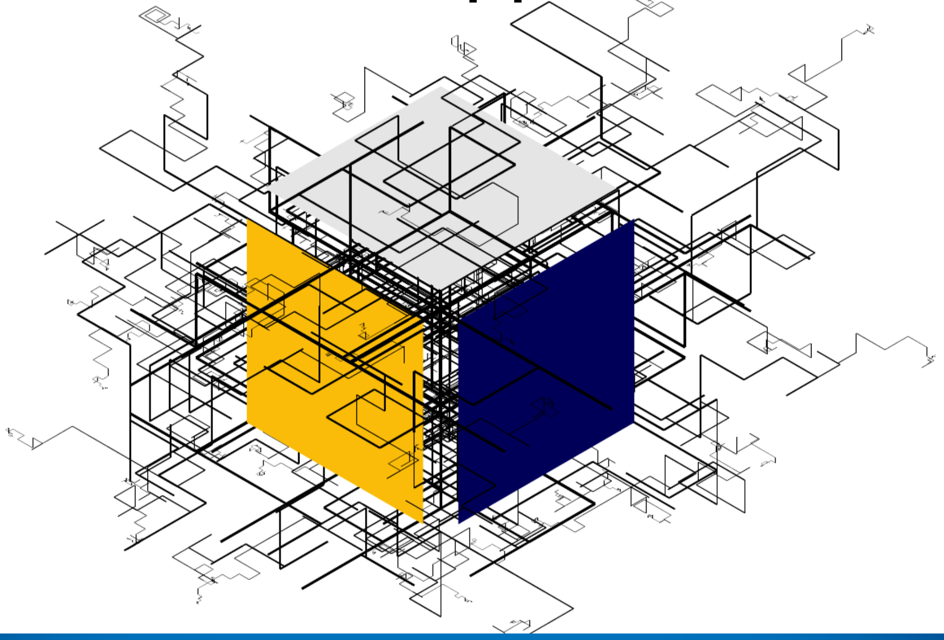
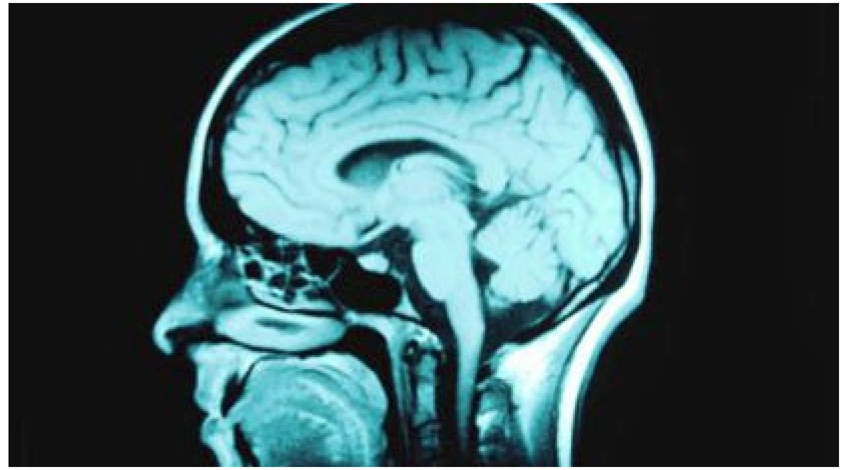
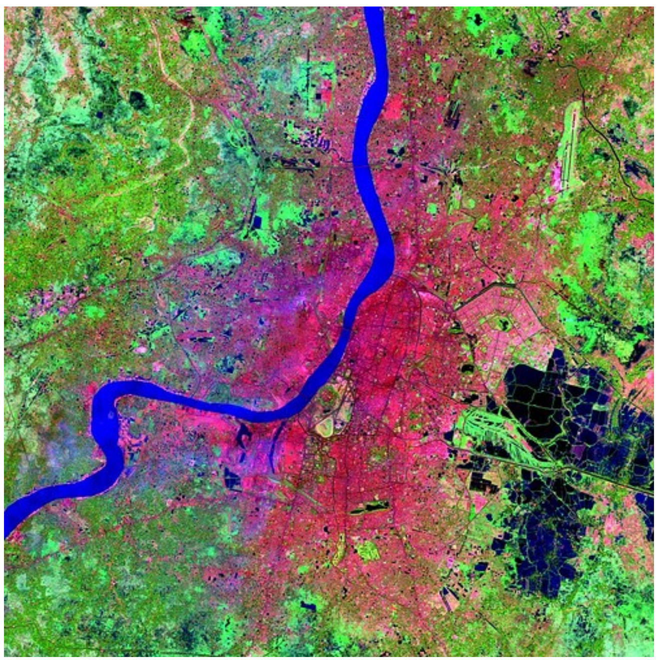
Assistant Professor, Applied Mathematics





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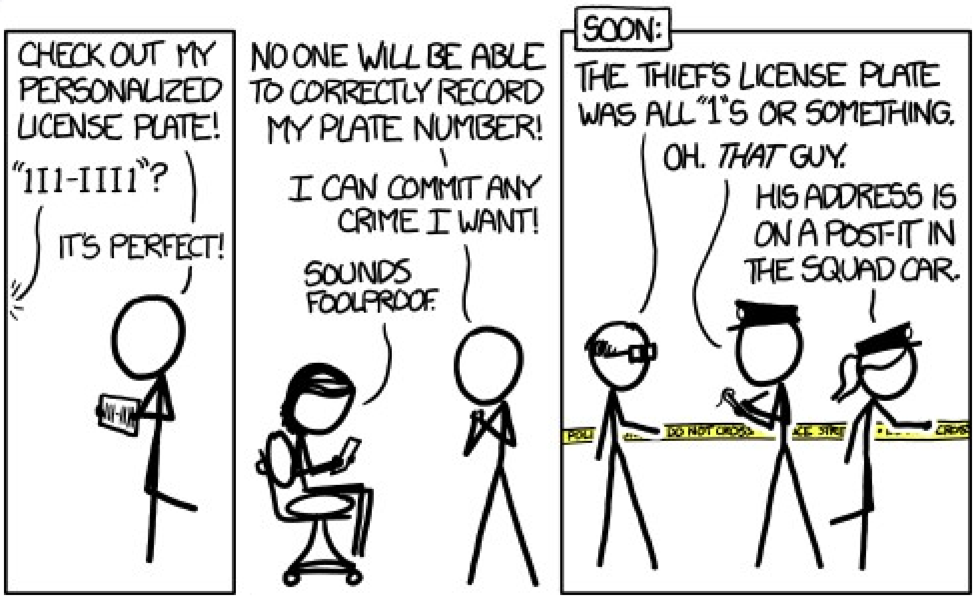
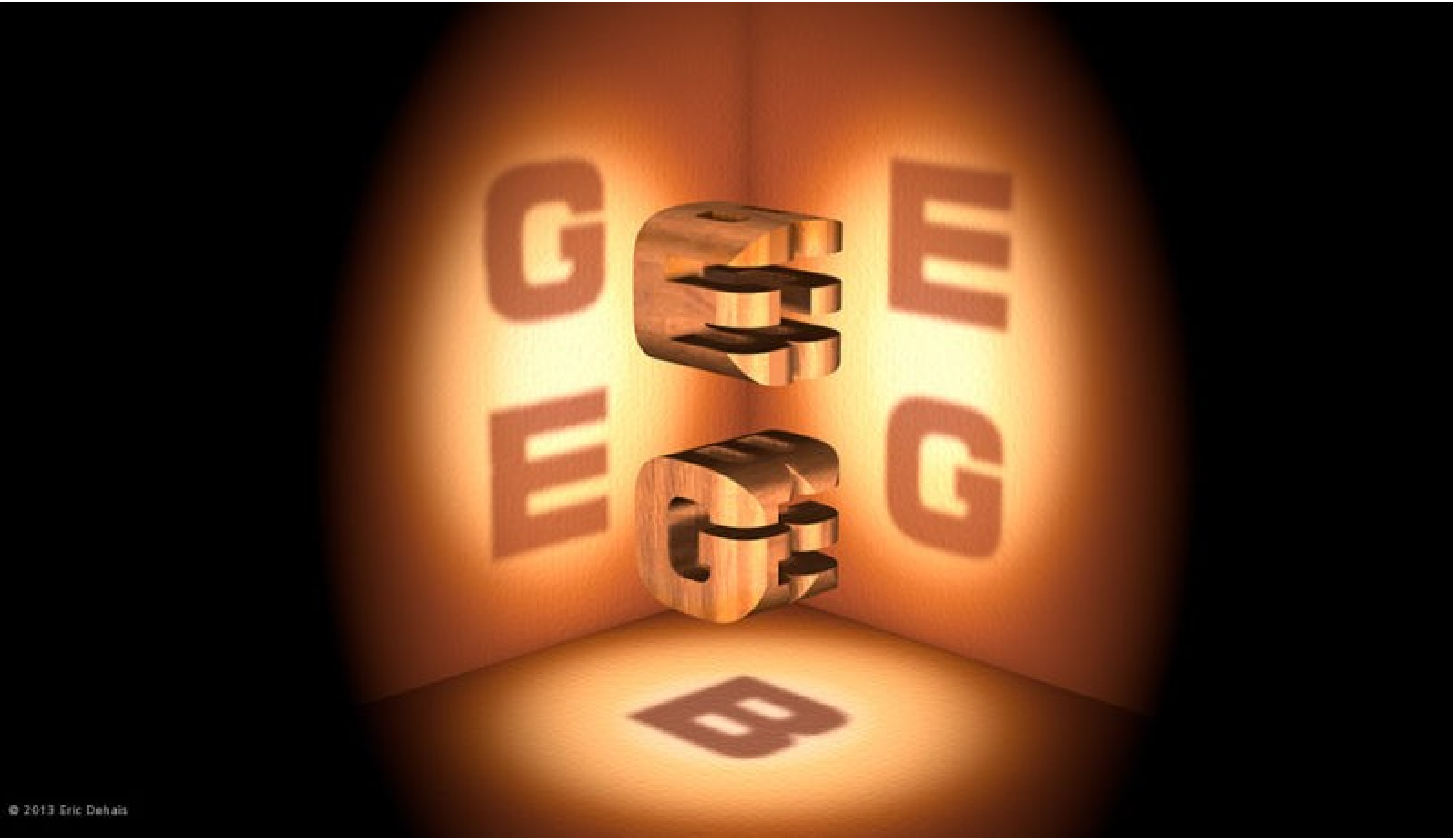

CHECK OUT MY PERSONALIZED LICENSE PLATE!
"111-1111"?
IT'S PERFECT!
NO ONE WILL BE ABLE TO CORRECTLY RECORD MY PLATE NUMBER!
I CAN COMMIT ANY CRIME I WANT!
SOUNDS FOOLPROOF.
SOON:
THE THIEF'S LICENSE PLATE WAS ALL "1"S OR SOMETHING.
OH. THAT GUY.
HIS ADDRESS IS ON A POST-IT IN THE SQUAD CAR.
DO NOT CROSS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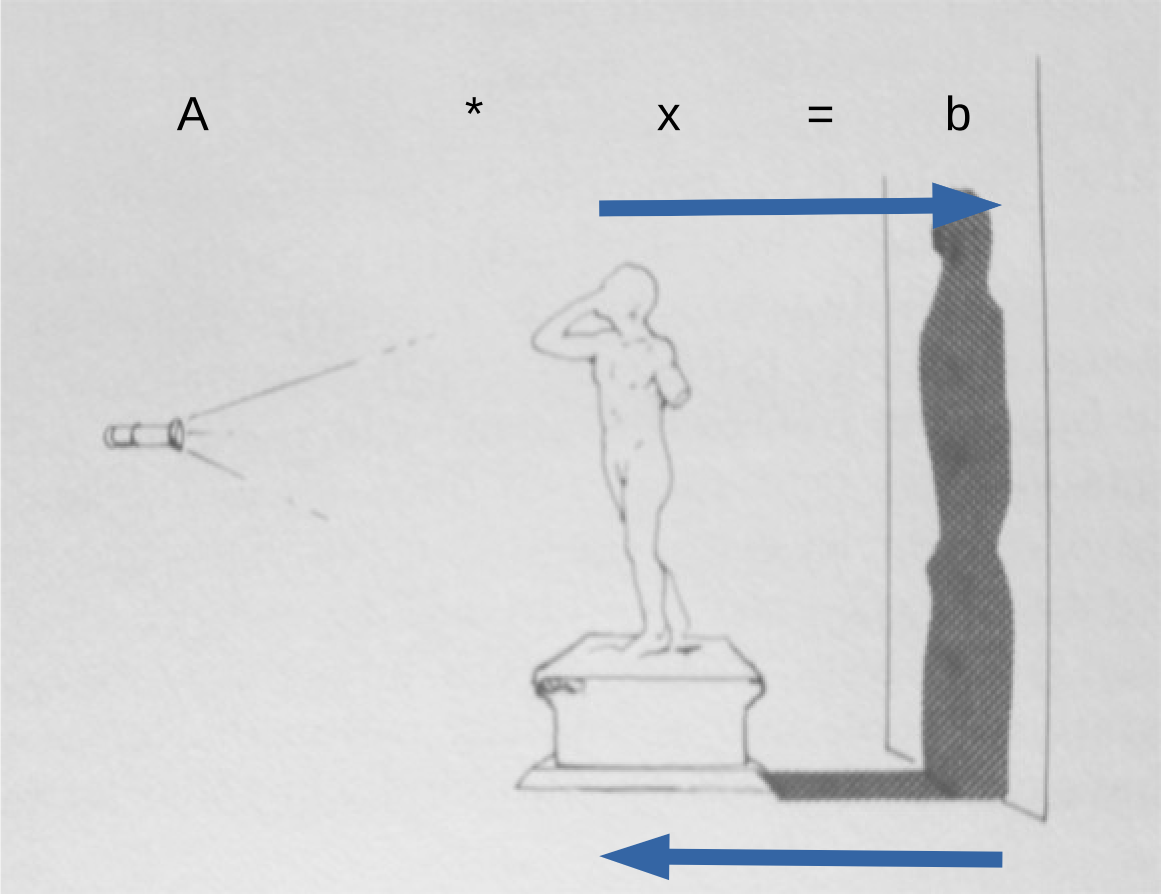


$b = a * x$

Forward problem

“easy”

$x = b / a$

Backward problem

“hard”

Ill-posedness:

1) solution might not exist,
2) solution might not be unique,
3) solution does not depend smoothly on the data

Variational method:

Find the “solution” **x that best explains the observed data**:

Minimize $|| Ax - b ||^2$

subject to a prior idea of what the solution should look like:

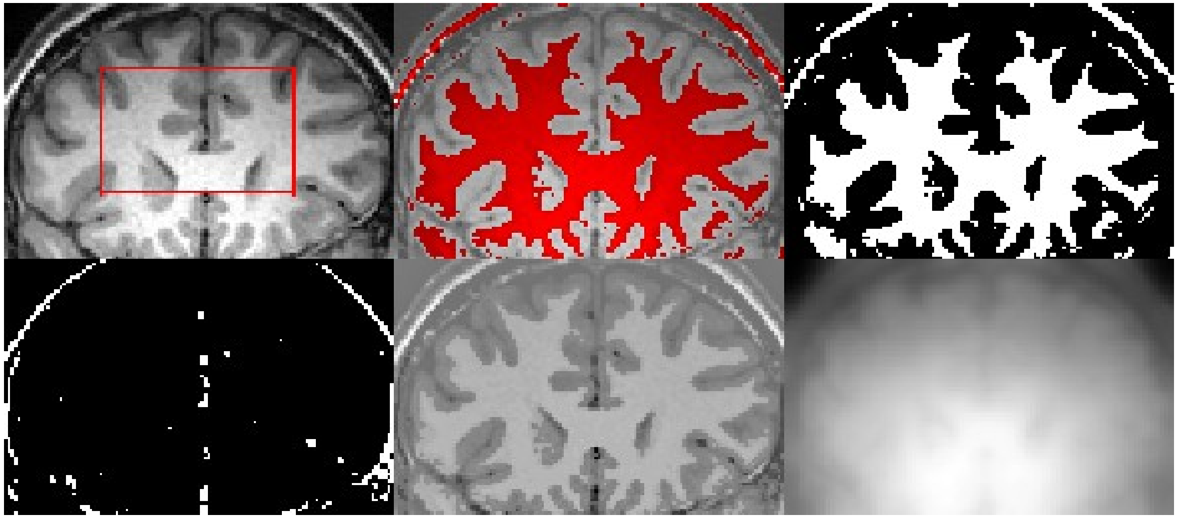
Minimize $|| Ax - b ||^2 \quad + \quad || Bx ||^2$

Occam’s razor:

Suppose there exist two explanations for an occurrence.
In this case the simpler one is usually better.

Ex: Image segmentation:

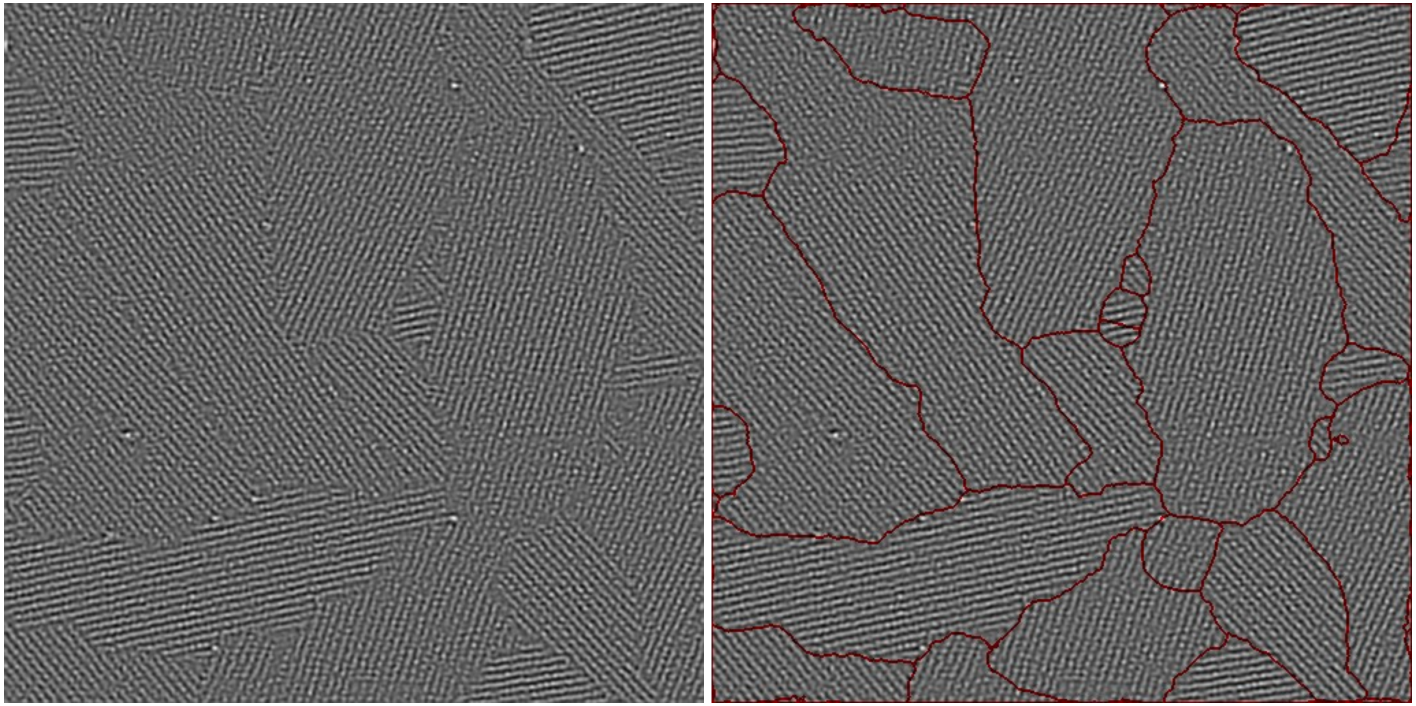
I=B+S u {0,1}



X {0,1} S (structure) B (bias)

$$E_{\mathrm{CVXB}}(\mu_1, \mu_2, u, X, B, S) := \lambda_1 \int_\Omega (1 - X)(\mu_1 - S)^2 u$$

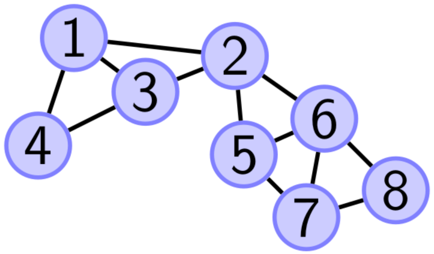
$$+ \lambda_2 \int_\Omega (1 - X)(\mu_2 - S)^2 (1 - u) + \beta \int_\Omega |\nabla u| + \gamma \int_\Omega X + \alpha \int_\Omega |\nabla B|^2 .$$

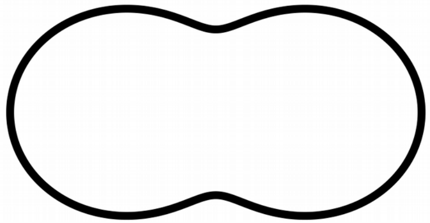


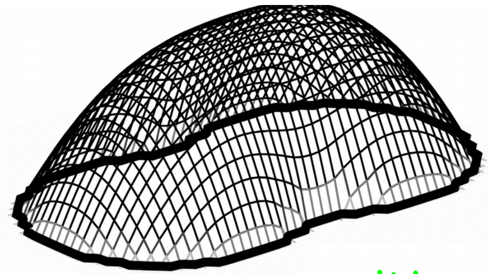
$$\min_{u_k:\,\mathbb{R}^n\to\mathbb{R},\ A_k:\,\mathbb{R}^n\to\{0,1\},\ \vec{\omega}_k\in\mathbb{R}^n} \left\{ \sum_k \left(\alpha_k \left\| \nabla \left[u_{AS,k}(\vec{x}) e^{-j\langle \vec{\omega}_k, \vec{x} \rangle} \right] \right\|_2^2 + \beta_k \|A_k\|_1 + \gamma_k TV(A_k) \right) \right\}$$

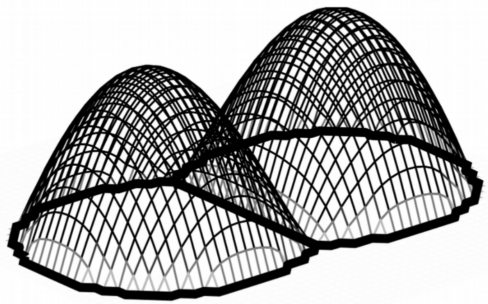
$$\text{s.t.} \quad \forall \vec{x} \in \mathbb{R}^n: \begin{cases} \sum_k A_k(\vec{x}) u_k(\vec{x}) = f(\vec{x}), \\ \sum_k A_k(\vec{x}) = 1. \end{cases}$$

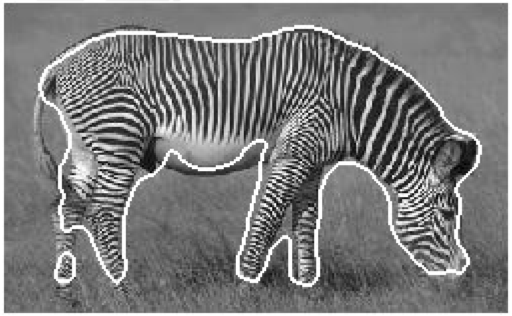
Ex: Graph partitioning











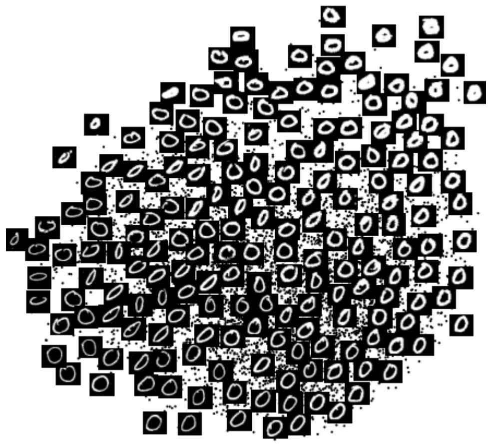
$$\min_{V=\sqcup_{\ell=1}^{k} V_\ell} \sum_{\ell=1}^{k} J^\star[V_\ell]$$

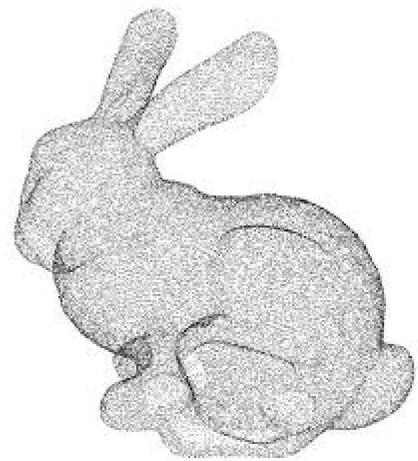
where

$$J^\star[S] := \min_u D[u] \quad \text{s.t.} \quad u\colon V \to \mathbb{R}, \ \|u\|_1 = 1, \ \text{and } u|_{S^c} = 0,$$

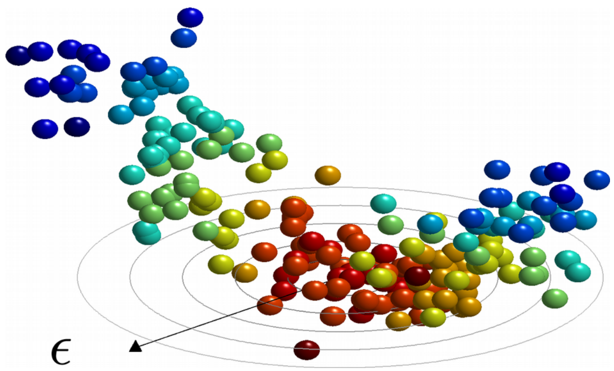
$$D[u] := \frac{1}{2}\|\nabla_w u\|^2_{L^2(E)} = \frac{1}{2}\sum_{(i,j)\in E} w(i,j)(u_i - u_j)^2,$$

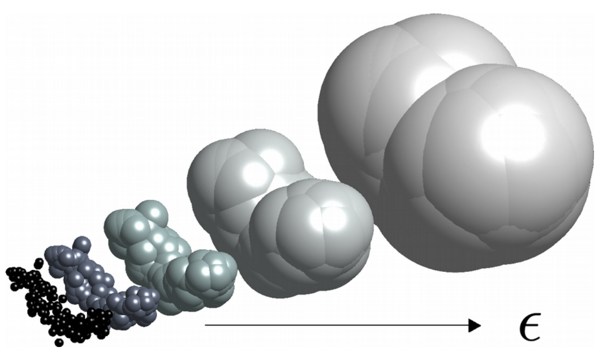
Graph-based Geometric Data Analysis

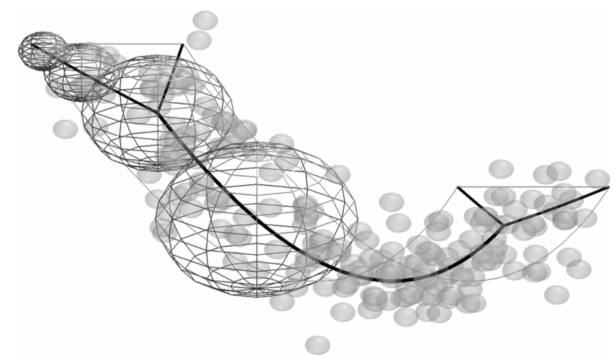




“ Graph theory is the new calculus. ”
(Daniel Spielman)







About:

- Myself
  - **PhD in EE**
    2011, EPFL Switzerland
  - **Postdoc / Visiting Prof.**
    2012-2016, UCLA
- **Catherine Potts**
  PhD student since 2016
- **Thomas Herndon**
  USP student AY 17/18
- **Summer and semester projects?**

- Relevant Courses
  - Machine Learning (M508)
  - Mathematical Imaging (M491/592 topics)
  - Numerical Methods (M441, M442)
- Requires:
  - Calculus & Diff. Equations
  - Linear algebra (M 221)
  - MATLAB, Python, …

Math faculty: Pure and Applied
(see also Statistics and Math Ed)

- Lisa Davis
- Jack Dockery
- Tomas Gedeon
- John Lund
- Scott McCalla
- Mark Pernarowski
- Jing Qin
- Tianyu Zhang
- Dominique Zosso

- David Ayala
- Lukas Geyer
- Ryan Grady
- Jaroslaw Kwapisz
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http://www.math.montana.edu