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# APPROVAL

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This writing project has been read by the writing project advisor and has been found to be satisfactory regarding content, English usage, format, citations, bibliographic style, and consistency, and is ready for submission to the Statistics Faculty.

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# An Overview of Accelerated Life Testing

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## Contents

# 1 Introduction to Accelerated Testing

Accelerated testing is accomplished by imposing stress on a product to cause it to fail faster than it would under normal operating conditions. At normal operating conditions, the time until product failure may be too long to allow effective predictions of the average life of a product. Therefore, a stress is imposed that accelerates the effects of use over time; failure time is a function of the stress factor. The data obtained under accelerated conditions are then projected back to normal operating conditions to predict the estimated life expectancy of the product. These studies ensure the quality and safety of products sold. [4]

Many industries perform accelerated testing on their products. For example, automakers will put vehicles in giant ovens heated to 130 degrees Celsius to increase part failure. Pipeline manufacturers apply stresses to pipes, valves etc. to see how long it takes the product to fail. Labs have used heat and vibration to simulate 50+ years of wear and tear on below ground storage tanks used at the Hanford Nuclear storage Facility. In just a few weeks they are able to mimic the existing conditions, and stop failures in the Hanford Nuclear storage Facility before they get worse. [3]

## 1.1 Acceleration Methods

There are three common ways of increasing the number of failures in an accelerated life test.

- Increase the use rate of the product. This method is used for products that are not used for prolonged periods. For example, the median life of a bearing for a washing machine is 12 years. This is assuming the washing machine is used 8 times a week. If the machine is tested at 112 loads per week, the median life is reduced to roughly 10 months. This assumes that the increase in the amount of usage does not change the relationship between number of cycles and failure. [3]

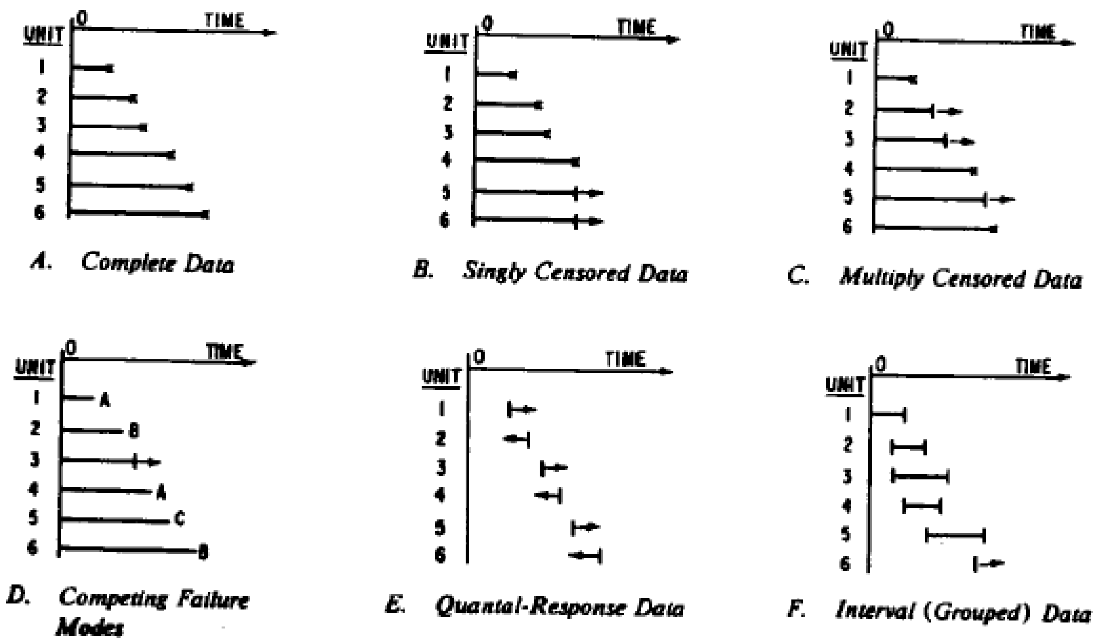
- Increase the intensity of the exposure to radiation. For example, organic materials will degrade when exposed to ultraviolet radiation. Electrical insulation exposed to gamma rays in nuclear power plants will degrade faster than it would normally. Increasing radiation intensity is similar to increasing use rate. [3]

- Increase the level of stress. For example, many machines are designed to operate in a certain range of temperatures. If the temperature range is increased (or decreased) more failures may occur. [7]

## 1.2 Censored Data

Life data are complete if the time to failure of each sample is known. However, special statistical models and methods are needed for failure time data due to complications imposed by censoring in the data. Censoring creates complications in obtaining estimates. The appropriate analysis is contingent on the data type. The data are classified as complete if the time to failure of each unit is known. A common occurrence is that the exact failure time of a unit is unknown.

For example, if a unit has not failed by the end of the study, then all that is known is that the failure would have occurred at some future point in time if the study had not been stopped. These types of data are called singly-censored. If a study has multiple stopping times, then such data are called multiply-censored. A unit that fails from multiple causes is classified as having competing failure modes. If a unit is only inspected once during a study, then the only information obtained is whether the unit failed before the inspection. These types of data are classified as quantal-response data. Finally, if units are inspected multiple times during a study all that is known is if a failure occurred between two inspection times. These types of data are termed interval or grouped data. [2]




**Types of Data (failed ×, survived ⊢, failed earlier ⊣ )**

Figure 1: Types of failure seen in accelerated life testing [6]

### 1.3 Overview of Planning an Accelerated Life Test

The following items describe methods for planning an accelerated life test [5]:

- Restrict testing to a range of levels of the stress factor that are within a plausible range of normal operating levels so that reasonable extrapolations back to normal conditions can be made. This will establish the maximum value for the stress factor used in the test.

- Select a medium level of the stress factor that is far removed from the maximum value so an estimated slope of the relationship between the stress factor and the amount of time until failure can be made. This medium value of the test factor should result in at least 10% of the number of failures within the test period.

- Finally, select a minimum value for the stress factor that is as close as possible to normal operating conditions.

- A large percentage of the testing units should be exposed to the lowest level of the stress factor. This approach of collecting most of the data within a close range to normal conditions will result in less extrapolation and thus better estimates for normal conditions. However, since the test covers a wide range of values, results of the relationship between the stress factor and time to failure are more adequately quantified.

## 2 Common Parametric Models

In reliability analysis, both parametric and non-parametric methods are used. However parametric models are used in accelerated testing because of the ability to extrapolate to lower and upper values in the distribution. Parametric models also have the advantage of being described with parameters instead of having to describe a whole curve and are strongly preferred by engineers for their ease of application and interpretability. [4]

Parametric accelerated life test models routinely have two components. There is an assumed distribution that describes the relationship between the life of the product at the varying levels of the stress factor. The Weibull and Lognormal distributions are appropriate for most applications. The second component that is quite common is a relationship between the distribution's parameter(s) and the stress variable. An example of this is the Arrhenius model that is used in describing the degradation process of a product exposed to high heat. The Weibull, Lognormal, and Arrhenius models will be discussed in the following sections. [4]

Models for life distributions help describe the product life times under normal conditions and accelerated conditions. The relationships between normal conditions and accelerated conditions are based on the failure mechanism. Collaboration between engineers and statisticians is essential because statisticians usually lack the insight needed to determine the relationship. However, for many products, there are standard models that describe the relationships that are applicable to most stress applications. The most common parametric models used in accelerated testing are the Lognormal model and the Weibull model. Both are flexible models that can be used to describe many types of data where the test units are statistically independent. Maximum likelihood estimation is utilized in estimating model parameters. [1]

## 2.1 Lognormal Model

The Lognormal distribution is used in metal fatigue tests, solid state components such as semiconductors, and electrical insulation. In many engineering applications log refers to base ten. However, the base chosen for the logarithm does not impact inferences as long as the base is kept the same through the analysis.[1]

The Lognormal PDF can be written as follows:

$$f(y) = \frac{1}{y\sigma\sqrt{2\pi}}=\mathrm{e}^{-[(log(y)-\mu)/\sigma]^2/2},\ y>0$$

$\mu$ is the log mean of life and is defined for all real numbers. $\sigma$ is the standard deviation of log life. The Lognormal CDF can be written as follows:

$$F(y) = \phi\Big(\frac{log(y)-\mu}{\sigma}\Big),\ y>0$$

The shape of the Lognormal distribution can take on many forms depending on the choice of $\mu$ and $\sigma$. [4]

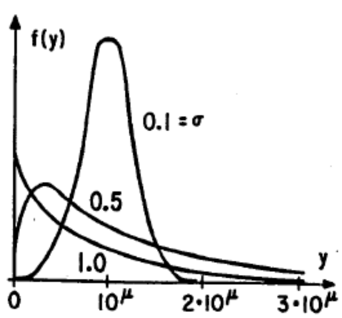



Figure 2: PDF Lognormal [6]

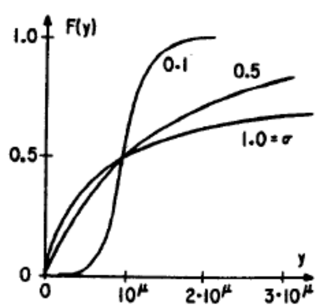



Figure 3: CDF Lognormal [6]

## 2.2 Weibull Model

The Weibull distribution is commonly used to model product life because it provides a simplistic model for increasing and decreasing failure rates. The Weibull distribution is capable of describing many types of failure distributions with a wide range of shapes. It is important to note that the Exponential distribution is the Weibull distribution with a shape parameter equal to one. Many engineering applications have used the Exponential distribution due to its simplicity. However, most product failures are more accurately modeled using a shape parameter other than one. Thus, the Weibull is a better alternative to the Exponential distribution. [4]

The two parameter Weibull PDF can be written as follows:

$f(y) = \dfrac{\beta}{\alpha^\beta} y^{\beta-1} e^{-(y/\alpha)^\beta}$, $y$ >0 having mean E(Y)=$\alpha\Gamma\left(1+\frac{1}{\beta}\right)$ and variance Var(Y)=$\alpha^2\left[\Gamma\left(1+\frac{2}{\beta}\right) - \Gamma^2\left(1+\frac{1}{\beta}\right)\right]$.

The Weibull CDF can be written as follows:

$$F(y) = 1 - e^{(-y/\alpha)^\beta},\ y > 0$$

By varying $\alpha$ and $\beta$ the Weibull distribution can take on many shapes that approximate many practical applications. [4]

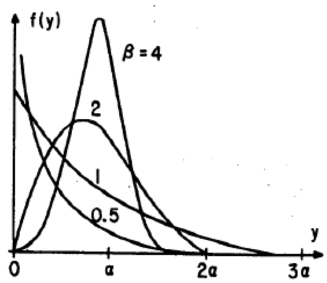



Figure 4: PDF Weibull [6]

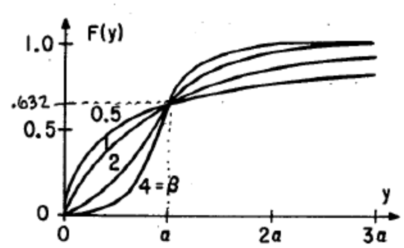



Figure 5: CDF Weibull [6]

### 2.2.1 Linearization

As can be seen by the graphs of the CDFs for the Lognormal distribution and the Weibull distribution, the distributions look very similar. Therefore, probability plots are useful tools as they have unique scales that plot distributions as a straight line. This allows assessing which distribution more adequately describes the data in addition to obtaining rough estimates of approximate failure times. [7]

The plot of $y$ opposite *F(y)* can be made linear by transforming *F(y)* and $y$ and thus linearizing the CDF [4]. The is commonly done by using the quantile function for *F(y)*. The quantile functions for both the Lognormal and Weibull distributions are as follows:

The Lognormal percentile is: $log(y_p) = \mu + \phi_p^{-1}\sigma$

The Weibull percentile is: $y_p = \alpha[-log(1-p)]^{1/\beta}$

# 3 Techniques to Analyze Accelerated Failure Time Data

A possible first step in assessing the appropriateness of a distribution is to plot a scatter plot of the failure time against the accelerating variable. However, if there is any censoring in the data it can be quite difficult to assess the relationship between failure time and the accelerating variable. Therefore, it is recommended a multiple probability plot of non-parametric CDF estimates at individual levels of the accelerating variable be created. Linearized forms of candidate distribution and their associated ML estimates can then be added to the plot. This allows graphical assessment of the most appropriate distribution for each level of the stress variable. [4]

An appropriate model is then fit to the data. Careful examination of residual plots should be conducted before inferences are drawn. If there is censoring, a plot of fitted values against standardized residuals can be difficult to interpret. Therefore, in the case of censoring, a plot of standardized residuals against the associated probability has an easier interpretation. This interpretation is the same as assessing the normality assumption when a QQ plot is used in normal linear regression. The difference here is that the distribution being assessed is not required to be normal. [4]

## 3.1 Arrhenius Model

In tests involving heat, the Arrhenius model often performs well and is commonly used in practice. Applications where it is used include: electrical components, solid state and semiconductor devices, battery cells, lubricants and grease, plastics, and incandescent lamp filaments. [7]

The Arrhenius model uses the Arrhenius Law which is defined as follows: The rate of a chemical activation depends on temperature where the relationship can be described as rate=$A'e^{[-E/(kT)]}$ [7];

E is the activation energy of the reaction, usually in electron volts.

k is the Boltzmann's constant, $8.6171 * 10^5$ electrons-volts per one degree Celsius.

T is the absolute Kelvin temperature. Kelvin temperature is defined as the Centigrade temperature plus 273.15 degrees.

$A'$ is a constant that is characteristic of the product failure mechanism and test conditions.

In the following example, the Lognormal model will be compared to the Arrhenius-Lognormal model. The Arrhenius-Lognormal regression model can be described as:

$$F(y) = \phi\Big(\frac{log(y) - \mu}{\sigma}\Big)$$

$\mu$=$\beta_0$+$\beta_1 x$, $x$=11605/temp(K), and $\beta_1$=E is the activation energy.

When a Lognormal model is fit, $\mu$ and $\sigma$ are estimated at each temperature without any constraints. The Arrhenius-Lognormal model estimates $\mu$ and $\sigma$ at each temperature and constrains $\mu$ to be a linear function of $x$=11605/(tempK); $\sigma$ is constrained to be the same for all temperatures. The likelihood of the Arrhenius-Lognormal model is always larger than the likelihood of the Lognormal model, but if the likelihood of the Arrhenius-Lognormal model is substantially larger, this indicates that the Arrhenius-Lognormal model does not fit the data as well as the Lognormal model. A likelihood ratio test with a $\chi^2$ test statistic is used to compare the two models. The test statistic used is $Q = -2(L_{Arr.Lognormal} - L_{Lognormal}) \sim \chi^2_3$. The three degrees of freedom is the difference in the number of parameters between the two models. [4]

# 4 Application to Electrical Motor Insulation Data

Excessive heat in motors can cause a number of performance problems. Overheating causes the motor wiring insulation to deteriorate quickly. [7]

Overheating occurs due to a number of factors. Every electric motor has a design temperature. If a motor is started at too high an operating temperature, problems can occur. Overheating also occurs when an electric motor is forced to operate in a high temperature environment. Electric motors must have a proper cooling system and a ventilation system; however, excessive temperatures can cause the systems that are used to dissipate heat to become over worked.

The data from Nelson [7] will be used to illustrate an accelerated life test for Class B electrical motor insulation. An electric motor's insulation system separates electrical components from each other, preventing short circuits and thus, avoiding burnout and failure. In motors, Class is defined by the maximum allowable operating temperature. Class B has a maximum allowable operating temperature of 130 degrees Celsius. Forty types of insulation were tested at 150, 170, 190, and 220 degrees Celsius. The median life of the insulation at the design operating temperature of 130 Celsius will then be estimated. Multiple observations were right censored indicating that the motor's electrical components did not short circuit by the end of the testing period.

## 4.1 Diagnostics and Model Fit

A multiple probability plot was fit to assess the appropriateness of the Arrhenius-Lognormal model vs. the Weibull model. The Arrhenius-Lognormal multiple probability plot has the Arrhenius-Lognormal ML fits for each temperature for the Class B data (Figure 5). Upon inspection, the Arrhenius-Lognormal model provides a slightly better fit to the data.

An Arrhenius-Lognormal model (constrained model) was compared to a Lognormal model (unconstrained model) using a likelihood ratio test. The likelihood for the constrained model

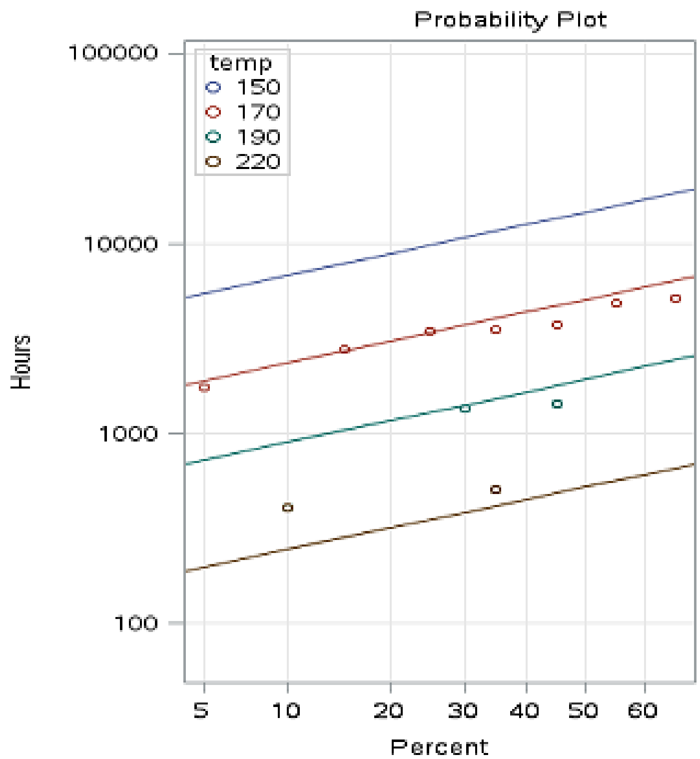



Figure 6: The Arrhenius-Lognormal multiple probability plot has the Arrhenius-Lognormal ML fits for each temperature for the Class B data.

is equal to -14.16. The likelihood for the unconstrained model is equal to -12.97. Therefore, $Q = 2.38 < \chi^2(.975; 3)$. This provides evidence that there is no inadequacy of the constrained model compared to the unconstrained model. Therefore, the Arrhenius-Lognormal model will be used over the Lognormal model.

Examination of residual plots was conducted. Because there is censoring, the plot of fitted values against standardized residuals is difficult to interpret. The appearance of the right downward slope is due to the right censored data. Therefore, a plot of standardized residuals against the associated probability was created. There does appear to be some deviation from linearity; however the deviations are not extreme and can be attributed to randomness in the data. Therefore, the Arrhenius-Lognormal model appears to be appropriate for this analysis.

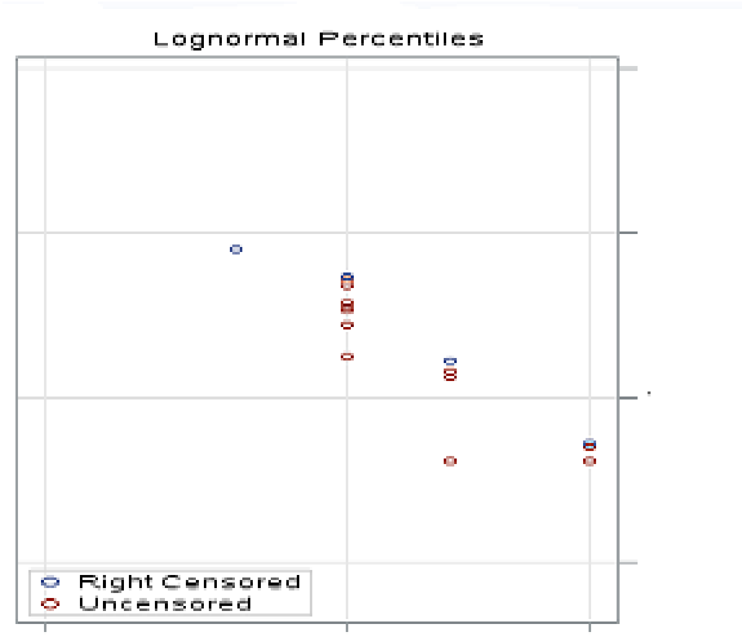



Figure 7: A plot of fitted values against standardized residuals.

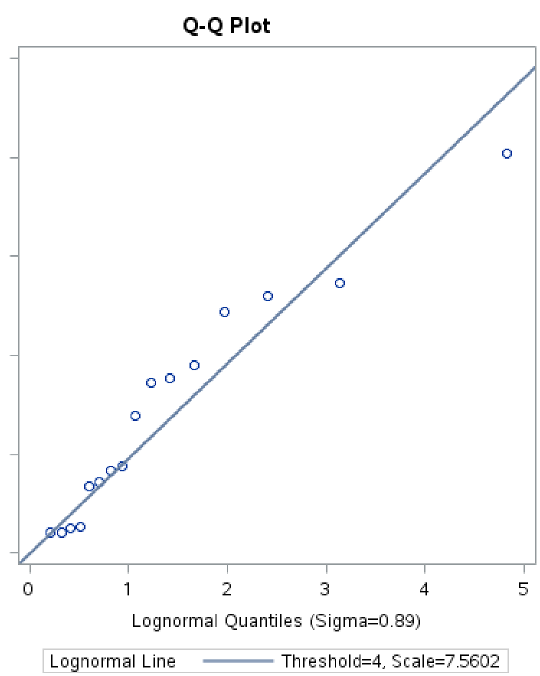



Figure 8: A plot of the standardized residuals against the associated probability.

## 4.2 Estimation for Normal Operating Conditions

The predicted median life at 130 degrees Celsius is estimated to be 34,241.71 hours with corresponding 95% confidence interval of 18,355.74 median lifetime hours to 63,876.19 median lifetime hours. This can be interpreted as under normal operating conditions, a motor's electrical components insulated with class B insulation will operate without short circuiting for approximately 34,241.71 hours.

| Observation Statistics | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Hours | censor | temp | count | Prob | Pcntl | Stderr | Lower | Upper |
| . | . | 130 | . | 0.1000 | 15350.862 | 4503.0803 | 8638.4943 | 27278.938 |
| . | . | 130 | . | 0.5000 | 34241.71 | 10892.946 | 18355.739 | 63876.192 |
| . | . | 130 | . | 0.9000 | 76379.732 | 30568.859 | 34858.48 | 167358.52 |

Figure 9:

# 5 Cautions of Accelerated Testing

The following discussion will address some of the potential problems of accelerated testing. The issues addressed here do not encompass all potential problems with accelerated testing. [4]

The application of stress can cause a test unit to no longer function properly in a way that would not occur under normal conditions. For example, the application of heat can cause a test unit to melt and thus "fail" whereas under normal conditions the test unit would not melt. This is why failure of a product needs to be well defined. If the product "fails" in a way that is not the failure type of interest, the test unit can be treated as a censored unit. Censoring can reduce the amount of information available to estimate the failure of interest. However, if test units not meeting the failure criteria are not censored, inferences from the study can be severely impacted.

In accelerated testing, results are extrapolated to values observed under normal conditions. Because there is extrapolation, there is also a lot of uncertainty in the estimates. Therefore, basing conclusions on a single point estimate is fraught with risk. It is recommended that inferences be made with regard to the plausible range of values reported in the estimate's confidence interval. It is imperative, however, that researchers understand that confidence intervals do not take into account model uncertainty. For this reason, sensitivity analysis is a recommended method for assessing the variability of estimates produced from multiple models.

In many cases, the failure types observed in accelerated testing can mask other important failure types; for example, failures that are seen in the field may rarely occur in the accelerated testing environment. Therefore, it is essential that tests are designed and performed with knowledge of the types of failures that occur in the field.

An increase in a stress variable can sometimes result in unexpected outcomes. For example, increasing temperature can actually decrease the number of failures observed. In this situation, there will be a much higher failure rate when the product is operated under normal conditions. Therefore, careful thought needs to be used in assessing what magnitude of stress will cause an increase in the observed number of failures.

The materials used in the test units must be the same quality as materials used in the field. Sometimes the quality of materials used in testing units differs from the quality of materials used in the field and this can result in poor conclusions. A common mistake is for a company to perform accelerated testing on a product and then manufacture the product with a lower quality material due to the cost of mass production.

# SAS Code Appendix

```
data project;
      input hours temp count censor;
      if _n_ = 1 then cntrl=1;
      else cntrl=0;
      label hours='Hours';
      datalines;
        .   130 . .
      8064 150 10 1
      1764 170 1  0
      2772 170 1  0
      3444 170 1  0
      3542 170 1  0
      3780 170 1  0
      4860 170 1  0
      5196 170 1  0
      5448 170 3  1
       408 190 2  0
      1344 190 2  0
      1440 190 1  0
      1680 190 5  1
       408 220 2  0
       504 220 3  0
       528 220 5  1
      ;
   run;

symbol v=plus;
title 'Lognormal Q-Q Plot for Diameters';
proc univariate data=project noprint;
   qqplot hours / lognormal10(theta=4 zeta=est sigma=.89
                               color=black l=2)
                     square;
run;
proc reliability;
   distribution lognormal10;
   freq count;
   model hours*censor(1) = temp /

      obstats( q=.1 .5 .9 control=cntrl );
   rplot hours*censor(1) = temp /
      pplot
      fit=model
      noconf
```

```
   relation = arr
   plotdata
   lupper = 1.e5
   slower=120;
run;
```

# 6 References


[1] Nikulin, V. and Bagdonavicius, M. (2002). Accelerated Life Models:
Modeling and Statistical Analysis. Chapman and Hall. 1st edition.

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