Brief Table of Laplace Transforms

$f(t)$	$F(s) = \mathscr{L}\{f\}(s)$
1	$\dfrac{1}{s}$
e^{at}	$\dfrac{1}{s-a}$
$t^n, n = 1, 2, \ldots$	$\dfrac{n!}{s^{n+1}}$
$\sin bt$	$\dfrac{b}{s^2+b^2}$
$\cos bt$	$\dfrac{s}{s^2+b^2}$
$e^{at}t^n, n = 1, 2, \ldots$	$\dfrac{n!}{(s-a)^{n+1}}$
$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$
$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$
$e^{at}f(t)$	$F(s-a)$
$f'(t)$	$sF(s) - f(0)$
$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$t^n f(t)$	$(-1)^n \dfrac{d^n}{ds^n}F(s)$
$(f * g)(t)$	$F(s)G(s)$
For $a \geq 0$,	
$f(t-a)u(t-a)$	$e^{-as}F(s)$
$g(t)u(t-a)$	$e^{-as}\mathscr{L}\{g(t+a)\}(s)$
$\delta(t-a)$	e^{-as}

Theorem 9.
If f has period T and is piecewise continuous on $[0, T]$, then the Laplace transform of one period, $F_T(s)$, is related to the Laplace transform by

$$F(s) = \frac{F_T(s)}{1 - e^{-sT}}.$$

If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible then $\mathbf{A}^{-1} = \frac{1}{|\mathbf{A}|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Variation of Parameters.

$$\begin{aligned}
\mathbf{x}_p(t) &= \mathbf{X}(t) \int \mathbf{X}^{-1}(t)\mathbf{f}(t)\,dt \\
\mathbf{x}(t) &= \mathbf{X}(t)\mathbf{c} + \mathbf{X}(t) \int \mathbf{X}^{-1}(t)\mathbf{f}(t)\,dt \\
\mathbf{x}(t) &= \mathbf{X}(t)\mathbf{X}^{-1}(t_0)\mathbf{x}_0 + \mathbf{X}(t) \int_{t_0}^{t} \mathbf{X}^{-1}(s)\mathbf{f}(s)\,ds
\end{aligned}$$

If $\mathbf{X}(t)$ is fundamental matrix for $\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t)$ then

$$e^{\mathbf{At}} = \mathbf{X}(t)\mathbf{X}^{-1}(0).$$

For any 2×2 matrix $\mathbf{A}$, the matrix exponential $e^{\mathbf{At}}$ can be computed according to the table below.

Eigenvalues of $\mathbf{A}$	$e^{\mathbf{At}}$
r_1, r_2 real and distinct	$e^{r_1 t}\frac{1}{r_1 - r_2}(\mathbf{A} - r_2\mathbf{I}) - e^{r_2 t}\frac{1}{r_1 - r_2}(\mathbf{A} - r_1\mathbf{I})$
r real repeated twice	$e^{rt}\mathbf{I} + te^{rt}(\mathbf{A} - r\mathbf{I})$
$\alpha \pm i\beta$ complex conjugate pair	$e^{\alpha t}\cos(\beta t)\mathbf{I} + \frac{1}{\beta}e^{\alpha t}\sin(\beta t)(\mathbf{A} - \alpha\mathbf{I})$