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Using Sampling Distributions to Build Understanding of Margin of Error

Introduction to Statistics

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1.1 Overview and Outline of Lesson

A goal of many statistical studies is to characterize interesting features of some population of interest (e.g., population mean, population variance). Although we can learn everything we would ever want to know about a population by examining its distribution, we usually cannot obtain the population distribution because we cannot observe every unit in the population. However, it may still be possible to learn something about the population by studying a subset (i.e., sample) of the population, as long as that sample is random. With some assumptions about the shape of the population distribution, sample statistics can be used to estimate population parameters with some margin of error. This lesson provides an opportunity for undergraduates to understand why we can use a sample mean (from a random sample) to estimate a population mean and how we can use the variability in a sampling distribution to help us estimate a margin of error.

1. Launch—Pre-Activity
   Prior to the lesson, undergraduates complete a Pre-Activity where they are presented with an example of a margin of error reported in the media and are asked to discuss why a margin of error might be reported.

2. Explore—Class Activity
   The context of the Class Activity focuses on investigating tests scores from a fake population of seniors at "Maplewood High School." Under a "fake world" premise where the population distribution is known, undergraduates work through the Class Activity to understand sampling variability in the context of using simulation to build an approximate sampling distribution for a sample mean. They then use the standard deviation of this sampling distribution (i.e., standard error) to build an understanding of margin of error.

   - *Problems 1 & 2*:
     Using 100 cards that represent the 100 test scores from a population of seniors at Maplewood High School, undergraduates engage in a hands-on simulation activity to get a feel for how sample means relate to the population mean. To do this, undergraduates randomly select samples of 10, compute the sample mean test score, create an approximate sampling distribution of sample means, and discuss sample-to-sample variability.

- *Problem 3*:
  Because technology provides a more efficient way of conducting a simulation, undergraduates use technology (e.g., StatKey, R, etc.) to conduct a simulation (instructors have access to the dataset of the 100 test scores). Undergraduates continue to simulate samples of size 10 and create an approximate sampling distribution of 500 sample mean test scores.
- *Problems 4–6*:
  Undergraduates explore two different methods (counting dots and the empirical rule) to identify the middle 95% of the sample mean test scores from their sampling distribution in Problem 3. Then, they use this information to construct a margin of error with a confidence level of 95% and relate this to $\pm 2 \times$Standard Error. In Problem 6, undergraduates consider when it is more convenient to use either the counting dots method or the empirical rule to construct a margin of error with a confidence level of 68% and 90%.
- *Problem 7*:
  Problem 7 concludes the activity and prompts undergraduates to write a sentence that uses a point estimate (e.g., sample mean) and a margin of error to report a range of plausible values in the context of test scores for seniors at Maplewood High School.

3. Closure—Wrap-Up
   Conclude the lesson by emphasizing the main points of the lesson. That is, if we have a random sample, we can use sample statistics to estimate population parameters with some margin of error. We can place a margin of error around our sample estimate to provide us with a range of plausible values.

1.2 Alignment with College Curriculum

A simulation-based introduction to inference using appropriate technology supports the development of statistical reasoning and can aid in student understanding of sample-to-sample variation and how to make inference using sampling distributions (see Carver et al., 2016; Bargagliotti et al., 2020). Deriving a margin of error through simulation in collegiate courses provides prospective teachers an opportunity to develop a deeper understanding of concepts they may teach their future students, such as sampling variability and its connection to margin of error. Further, the use of simulation models may help all undergraduates better understand concepts like bootstrapping, which may appear later in your course.

Incorporate this lesson into your curriculum as you see fit. This lesson emphasizes the use of simulation to construct sampling distributions, which are used to build an intuitive understanding of margin of error; we conclude the lesson by indirectly asking undergraduates to find a range of plausible values. Although we do not explicitly refer to this range of plausible values as a confidence interval, a margin of error is directly tied to a desired confidence level so this lesson can naturally lead into a lesson about confidence intervals.

1.3 Links to School Mathematics

High school students are expected to conduct simulations to explore, describe, and summarize sample-to-sample variability of a statistic; that is, create an approximate sampling distribution of a statistic. This lesson addresses several statistical knowledge and mathematical practice expectations recommended by professional organizations (e.g., Common Core State Standards for Mathematics [CCSSM, 2010]; Pre-K–12 Guidelines for Assessment and Instruction in Statistics Education II [GAISE II; Bargagliotti et al., 2020]). For example, high school students are expected to understand that sample statistics (from a random sample of a population) can be used to estimate population parameters (c.f. CCSS.MATH.CONTENT.HSS.IC.A.1 and CCSS.MATH.CONTENT.7.SP.A.1). Further, they are expected to use simulation models to create an approximate sampling distribution and construct a margin of error (c.f. CCSS.MATH.CONTENT.7.SP.A.2 and CCSS.MATH.CONTENT.HSS.IC.4).

This lesson highlights:

- Making inferences about a population parameter based on a random sample from that population;
- Conducting a simulation to create an approximate sampling distribution;
- Constructing a margin of error using simulation models.

1.4 Lesson Preparation

Prerequisite Knowledge

Undergraduates should know:

- The difference between a population and a sample;
- How to create a dotplot and calculate summary statistics such as a sample mean;
- The empirical rule.

Learning Objectives

In this lesson, undergraduates will encounter ideas about teaching mathematics, as described in Chapter 1 (see the five types of connections to teaching listed in Table 1.2). In particular, by the end of the lesson undergraduates will be able to:

- Use data from a random sample to estimate a population mean;
- Conduct a simulation to create an approximate sampling distribution and to construct a margin of error;
- Write a sentence that uses a point estimate (e.g., sample mean) and a margin of error to report a range of plausible values in the context of a problem;
- Examine hypothetical students' understanding of margin of error and evaluate questions to help guide students' statistical understanding about reporting a margin of error and using different methods for estimating a margin of error from a unimodal and symmetric sampling distribution.

Anticipated Length

Two 50-minute class sessions.

Materials

The following materials are required for this lesson.

- Pre-Activity (assign as homework prior to Class Activity)
- Class Activity (print Problems 1–2, 3, 4–6, and 7 to pass out separately)
  - 100 Test Score Cards (cut out cards and place in a bag for each group to use for Problems 1–2)
  - 100 Test Scores Dataset (.csv file to share with undergraduates when they conduct a computer simulation)
  - Appropriate technology and software for the instructor and undergraduates to conduct a simulation and create an approximate sampling distribution.
- Homework Problems (assign at the end of the lesson)
- Assessment Problems (include on a quiz or exam after the lesson)

All handouts for this lesson appear at the end of this lesson, and LaTeX files, along with the .csv file of Test Scores, can be downloaded from INSERT URL HERE.

1.5 Instructor Notes and Lesson Annotations

Before the Lesson

Assign the Pre-Activity as homework for undergraduates to complete in preparation for the lesson, and ask undergraduates to bring their solutions to class on the day you start the Class Activity.

Pre-Activity Review (5–10 minutes)

As a class, discuss the solutions to the Pre-Activity.

Problem 1 illustrates how a margin of error (ME) is commonly reported in the media (i.e., Point Estimate $\pm$ ME), and by the end of the Class Activity, undergraduates will write a similar type of sentence that uses a sample mean and a margin of error to report a range of plausible values in the context of a problem.

Pre-Activity

1. Consider the following sentence from a statistical report, as presented in De Veaux, Velleman, and Bock (2012).

   Based on meteorological data for the past century, a local TV weather forecaster estimates that the region's average winter snowfall is 23 inches, with a margin of error of ± 2 inches.

   (a) If you lived in this region, would you want the margin of error to be large or small? Explain.

   Sample Responses:
   - I would want the margin of error to be small so that I could budget my snow removal costs more accurately. If there is a larger margin of error, it would be much more difficult to estimate how much I might need to pay for snow removal throughout the winter.
   - I'd want the margin of error to be small so that the estimate of "23 inches" is more likely to occur during the winter snowfall.

   (b) Why do you think a margin of error is reported?

   Sample Responses:
   - I think the margin of error is reported to show how much average variation there is in the amount of snow that falls each winter. If there is larger variation, you might have no snow at all or way more snow, so the 23 inch estimate would not be very helpful.
   - It's reported to illustrate the reliability of the "23 inches of snowfall" as a year-to-year expectation.
   - To give a sense for how much variation we can expect around 23 inches of annual snowfall in the region.

   Commentary:
   From our experience, it is common for learners to misinterpret the word "error" in margin of error and assume it means something is "wrong" (e.g., a measurement error). Address how the word "error" can be misleading and let undergraduates know that a margin of error does not represent some sort of "mistake" in the data collection process. Rather, it refers to "estimation error," which is directly related to sampling variability (or the variability of an estimator, when a random sample from a single population is assumed).

Conclude the Pre-Activity Review by letting undergraduates know that the purpose of this lesson is to use simulation models to develop an understanding of margin of error and discuss the following connection to teaching.

Discuss This Connection to Teaching

National and state content standards now emphasize statistics content standards to be integrated throughout grades 6–12. Within a high school intermediate algebra course, students are expected to construct a margin of error through the use of simulation models for random sampling. The goal of this lesson is to show undergraduates how they can conduct a simulation to construct an approximate sampling distribution and build up to a margin of error.

Class Activity: Problems 1 & 2 (15–20 minutes)

Pass out **Problems 1 and 2** of the Class Activity, and ask undergraduates to work in small groups. See Chapter 1 for guidance on facilitating group work and selecting and sequencing student work for use in whole-class discussion.

Introduce the purpose of these two problems by letting undergraduates know that

- The dotplot contains 100 test scores, which represents the entire population of test scores for seniors at Maplewood High School.

- Our population consists of 100 seniors and we are assuming that we know the population mean test score. Typically we do not have access to data from an entire population and thus do not know the population mean. However, for this activity we are going to assume we know what the population mean is so we can get a feel for how sample means relate to the population mean, characterize variability in the sampling process, and develop an understanding of how to construct a margin of error.
- In this activity, we are going to use simulation to take many random samples from the same population and compute each sample's mean. From this, we will look at the overall patterns of the sample means and see how they relate to the population mean.

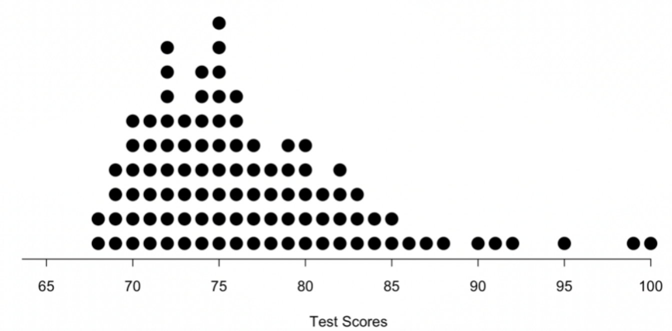
When a study in the real world is designed to learn about a population mean, we typically use the sample mean to estimate the population mean. In this lesson, undergraduates will build intuition about why the sample mean is a useful estimator of the population mean (when it is computed from a random sample). To help undergraduates build this intuition, we are applying a "fake world" context in this activity and assuming we know the population mean. This allows undergraduates to compare their sample means to the population mean and see how a sample mean from a random sample is usually a good guess of the population mean. Having each student (or small groups of students) take their own sample and compute their own sample mean also showcases the variability in the sampling process. Some samples will produce sample means very close to the population mean, while others will not. In the real world, we won't know if we have a sample mean "far from" or "close to" the true mean, so this motivates our need to understand how much variability we can expect in the sample means (of a fixed size from a fixed population).

We were intentional about generating this fake dataset in a way to not distract from concepts that build an understanding of margin of error. We chose a context that is straightforward to understand and created a dataset with exactly 100 data points whose values are "nice" in the sense that they are whole numbers, making all computations and methods for computing a margin of error as straightforward as possible.

Context for Class Activity

Investigating Test Scores for Seniors at Maplewood High School

Suppose that our entire population is 100 student test scores taken from the seniors at Maplewood High School (see the *population distribution* below), and we know the population mean test score (out of 100 points) is $\mu = 76.96$ points.



Before beginning Problems 1 and 2, ask undergraduates to briefly comment on the shape, center, and spread of the population distribution because they will make comparisons to this population distribution in Problem 3. Give each group a bag containing the 100 cards that represent the population of test scores, and demonstrate how to randomly draw a sample of size 10, as needed. Remind undergraduates to return the cards back to the bag (and to shuffle the cards within the bag) after they finish randomly selecting 10 of the cards. For Problem 1, focus the discussion on the sample-to-sample variation that is present among each group's sample of 10 test scores.

Class Activity Problem 1

1. You have a set of 100 cards that represent the 100 test scores from the entire population of seniors at Maplewood High School. Randomly draw a sample of **size 10** from these cards.

   (a) Write down the 10 test scores you randomly selected and compute the mean test score of your sample.

   Sample Response:
   Answers will vary. One sample is

   $$80, 72, 70, 82, 76, 79, 69, 74, 76, 83$$

   which has a mean of 76.1 points.

   Commentary:
   Make sure undergraduates record their sample mean on the Class Activity because they will use this value as their point estimate on Problem 7, where they will be prompted to write a sentence that uses a point estimate and a margin of error to report a range of plausible values in the context of the typical test score for seniors at Maplewood High School.

   (b) Share your sample mean test score with classmates and compare. Why do you think your sample mean test score is different from others?

   Sample Responses:
   An ideal response will include the concept of sample-to-sample variability.

   - My sample mean is different from others because of random chance. Different test scores may be selected when a different random sample is taken.
   - Our sample means are different because we did not all draw the exact same 10 test scores from our bags.

   Commentary:
   From our experience, some undergraduates may incorrectly explain that differences occurred as a result of having a "bad" sample or as a result of calculating the sample mean incorrectly. If this occurs, let undergraduates know that sampling variability is a characteristic of the real world, and it's not an indication of a "bad" sample or a computation mistake. The fact that not all random samples of 10 test scores are the same tells us that we need to expect some variation in our sample means. A single point estimate, like the sample mean computed in part (a) does not tell the whole story. We want to know "how good" this estimate is, and one way to report the quality of an estimate is to report the sample-to-sample (or sampling) variability. One way to convey the sampling variability is to report a range of plausible values.

In Problem 2, undergraduates continue to draw samples of size 10 from their bag of cards. Encourage undergraduates to take turns drawing samples from the bag. As undergraduates work on this problem, set up a way for you to compile the class's sample means so that you can display a dotplot of their sample means (i.e., an approximate sampling distribution of sample means). Alternatively, you may consider passing out 10 sticky notes to each group and asking them to write a sample mean test score on each sticky note. Then ask undergraduates to appropriately place their sticky notes on the board. This may help them visualize their "dots" in the sampling distribution.

Class Activity Problem 2

2. What happens if we repeat this process? From your set of 100 cards, continue to randomly draw a sample of size 10 and compute the mean test score of your sample. Repeat this process until you have a total of 10 sample means.

(a) Write the 10 sample mean test scores below.

Sample Response:

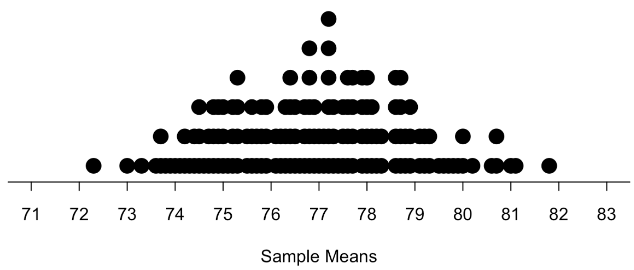
Answers will vary. Ten sample means (units = points) include:

$$76.1, 75.9, 75.9, 75.8, 76.4, 75.2, 76.6, 77.3, 79.7, 78.8$$

(b) As a class, create a dotplot of everyone's sample means and sketch it below. This is the class's *approximate sampling distribution* of sample mean test scores. Describe what one dot in the dotplot represents.

Solution:

Below is an example of a dotplot we created from 150 sample mean test scores. The mean of this approximate sampling distribution is 76.9 points and the standard deviation, referred to as the standard error, is 1.9 points.



The sample responses below describe what one dot in the dotplot represents:

- 100 cards were marked with test scores to represent the scores in the population. I sampled 10 of them and calculated the mean test score from that sample. The dot represents this sample mean.
- A dot represents the mean of one sample where 10 test scores were randomly selected.

Commentary:

Emphasize that you created an *approximate* sampling distribution of the sample mean test scores. We call it an approximate sampling distribution because it was constructed from roughly 150 sample means (exact number depends on how many groups of students you have in your class), and it was found empirically. True sampling distributions would include means from all possible samples of size 10 from the population of 100 test scores (i.e., the sample means resulting from the $\binom{100}{10}$ potential samples of size 10). "In real life, we get only one sample, but the sampling distribution gives us a mechanism for asking what would happen if we could take a random sample again and again" (Peck et al., 2013, p. 68). Simulation can be used to find approximate sampling distributions which provides us with a pretty good idea of what the true sampling distribution would look like.

"What is one dot?" is a challenging but useful question for undergraduates to think about. Describing what one dot represents will help undergraduates understand what the computer simulation is doing for them in Problem 3. If undergraduates are struggling with this question, ask them to find their dot on the sampling distribution and to describe what they did by hand with the cards to obtain the value of that dot.

After Problem 2, discuss the concept of a sampling distribution by sharing the following connection to teaching.

Discuss This Connection to Teaching

Sample-to-sample variation (or sampling variability) is a "big idea" concept in statistics (see Peck et al., 2013). The sampling distribution of a sample statistic (such as a sample mean) describes how sample statistics vary from one sample to the next. The sampling distribution of a sample statistic allows us to answer the following questions:

- How much will the value of a sample statistic tend to differ from one random sample to another?
- How much will the value of a sample statistic tend to differ from the corresponding population value? (Peck et al., 2013, p. 36–37)

The concept of a sampling distribution is fundamental to teaching and learning statistics, and implementing problems such as Problem 1 and 2 in the classroom is a way to allow students to notice that everyone's samples are similar but different. Further, by constructing an approximate sampling distribution, students can see sampling variability in the sample mean. All undergraduates will benefit from visualizing this sampling variability, and this is especially true for prospective teachers who will teach their future students about this big idea concept in statistics.

Class Activity: Problem 3 (20 minutes)

Pass out **Problem 3** of the Class Activity and instruct undergraduates to continue working in their small groups. Each group will need access to a computer and the dataset of 100 test scores so they can conduct a computer simulation. Instruct undergraduates on how to use your program of choice (e.g., StatKey, R) to conduct the simulation. As undergraduates use technology to build a sampling distribution, remind them to think about what they just did by hand and how it relates to what the computer is doing to conduct the simulated sampling distribution.

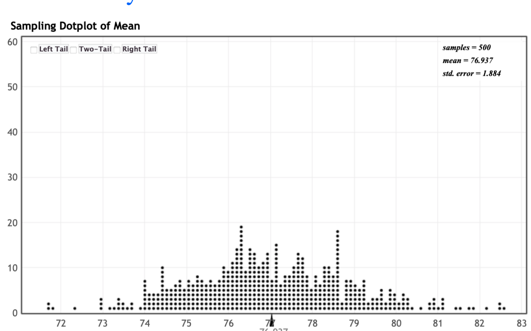
Class Activity Problem 3 : Parts a & b

We can continue to perform a simulation by hand with the cards, but technology provides us a more efficient way to do this! Our goal is to see what other sample means we might have gotten from different samples of size 10, and simulation is a tool we can use to see how variable the sample mean test scores might be.

3. Using the population of 100 test scores, use technology to conduct a simulation. Randomly draw a sample of **size 10** and compute the mean test score of your sample. Repeat this process for a total of 500 times.

   (a) Create a dotplot of your 500 sample mean test scores (i.e., another approximate sampling distribution) and sketch it below. Describe what one dot in the dotplot represents.

   Sample Response:
   Below is an example of a sampling distribution that was created in StatKey. The mean is 76.937 points and the standard error is 1.884 points. A single dot represents the mean of one sample where 10 test scores were randomly selected.



Commentary:

- Check that undergraduates' sampling distributions are reasonable—their sampling distribution should be centered around 76.96 (the population mean).
- We have found it useful to ask undergraduates to take a screen shot of their sampling distribution because they will continue to use this sampling distribution to answer Problems 4 and 5.

(b) Describe the shape, center, and spread of your sampling distribution from Problem 3(a). What is the mean and standard deviation of your sampling distribution? Note that the standard deviation of a sampling distribution is referred to as a *standard error*.

Sample Response:

- Shape = approximately unimodal and symmetric
- Center = 76.937 points (mean of the sampling distribution)
- Variability = 1.884 points (standard deviation of the sampling distribution)

Commentary:

Explain that the standard deviation of a sampling distribution is called a *standard error*. Emphasize that standard error is not an "error" in the sense of measurement error, but instead standard error represents the sample-to-sample variation in a population for a sample of size n. The standard error provides us with an estimate of the variability in the sample means that is due to the sampling process alone. That is, it provides us with a way to quantify how much background noise we can expect to see when taking a sample mean from a population (i.e., a guess at how much the value of a sample mean will differ from across samples). This is an essential piece of information for statistical inference because it allows us to provide a range of values instead of just one "best guess."

Problems 3(c) and 3(d) guide undergraduates to compare the approximate sampling distribution to the population distribution and to recognize that the mean of the approximate sampling distribution is centered near the population mean. Because the center of the sampling distribution is generally very close or at the population mean, the sample mean is an unbiased estimator of the typically unknown population mean. After discussing the solutions with the class, emphasize when a sample statistic can be used to estimate a population parameter (i.e., we have seen that when the sample is a random sample from the population of interest, it is often a good guess (i.e., "close to" the population mean). Though we won't know for certain if it's a good guess or a bad guess, we can use the standard error to help us understand how much variation there is between good and bad guesses in a given population.

Class Activity Problem 3 : Parts c & d

(c) Compare the shape, center, and spread of your sampling distribution from Problem 3(a) to the population distribution presented on page 1 of the Class Activity.

Sample Response:

- The population distribution and the sampling distribution are both unimodal. However, the population distribution is skewed right and the sampling distribution is unimodal and symmetric.
- The center of these two distributions is almost the same!
- The sampling distribution has smaller variability than the population distribution. The data in the population distribution range from about 67 points to 100 points but the data in the sampling distribution range from about 72 points to 82 points.

(d) Based on your sampling distribution from Problem 3(a), is a sample mean (computed from a random sample of size 10) a good way to estimate the population mean test score? Explain your reasoning.

Sample Responses:

- Yes, because most of the sample means in the sampling distribution are near the population mean of 76.9 points.
- We can see that a sample mean computed from a random sample of the population most often results in a sample mean that is close to the true population mean (i.e., the center and most of the mass of the sampling distribution is at or near the population mean).

Commentary:

- This is an important question. Although there were some samples that had sample means far away from the true population mean, most of the samples had sample means that were close to the true population mean.
- You may need to remind undergraduates that they are still working under the fake premise of knowing the population distribution. Further, the "sampling distribution gives us a mechanism for asking what would happen if we could take a random sample again and again" (Peck et al., 2013, p. 68). In real-life, we often get only one sample and if it was a random sample, we can use that sample mean as our "best guess" at the true population mean, and we can use the standard error to quantify how much variability we'd expect to see from sample mean to sample mean.

After discussing the solutions to Problem 3, emphasize the following connection to teaching, which focuses on the use of simulation-based method to teach and learn statistics.

Discuss This Connection to Teaching

High school teachers are expected to implement a simulation-based introduction to inference in their classes; their students will use data collected from a random sample and simulations to make inferences about a population (Peck et al., 2013). Prospective teachers need to be familiar with simulation as an instructional tool to develop their future students' conceptual understanding of many statistical ideas and to address expectations from content standards. For example, using a simulation to learn about sampling distributions and a margin of error provides students an opportunity to think about the process rather than trying to interpret theoretical approaches (Burrill, 2021). In addition, conducting a simulation by hand first and then following up with a computer simulation is a good pedagogical technique to use as students can connect what the computer is doing to what they did by hand.

Advice on Delivering the Lesson Over Two Class Sessions

If you are teaching this lesson over two class sessions, stopping around Problem 3 is a good place. See Chapter 1 for guidance on using exit tickets to facilitate instruction in a two-day lesson.

Discussion: Margin of Error and Range of Plausible Values (15 minutes)

The remainder of the Class Activity shifts its focus to building understanding of margin of error and writing a range of plausible values for a population parameter that we note are associated with different confidence levels. In this lesson, we do not spend time defining confidence or interpreting these ranges of plausible values as confidence intervals. We have found that instructors often follow this lesson with a lesson about confidence intervals and their interpretation.

Problems 4 and 5 do not rely on the standard, formal definition or formula of the margin of error that is commonly used when you are given only one sample. Because we are still operating under the fake premise of knowing the population distribution and being able to repeatedly sample from that population distribution, these problems prompt undergraduates to use their sampling distribution to construct a margin of error. Further, these problems focus on two methods ("counting dots" and the empirical rule) to understand the connection between the middle 95% of the sample means and a margin of error with a 95% confidence level. We leave it to the instructor to decide when they want to define the formal definition and formula of the margin of error; some teachers have found it works well to discuss the formula after this lesson and under the premise that we typically work with one random sample from an unknown population distribution.

Discuss This Connection to Teaching

Constructing a margin of error through the use of simulation models for random sampling is an explicit high school content standard that prospective teachers will be expected to teach.

Class Activity: Problems 4–6 (25 minutes)

Pass out **Problems 4–6** of the Class Activity, and instruct undergraduates to continue working in their small groups. Problem 4 focuses on "counting dots" to build a margin of error while Problem 5 focuses on using the empirical rule to build a margin of error. We intend that undergraduates will recognize that these two methods produce a similar answer for a margin of error with a confidence level of 95% because their sampling distribution is unimodal and symmetric.

Class Activity Problem 4

In Problem 3, we found that a sample mean (computed from a random sample) is usually a really good guess of the population mean. But, not all good guesses are created equally! We also saw in that sampling distribution that some of our good guesses were further from the population mean than others. In statistics, we care about how much variability is present among these sample means (estimated by the standard error you found in Problem 3), and we often report a range of plausible values for a population mean. Problems 4 and 5 guide you through two methods for estimating a margin of error from a unimodal and symmetric sampling distribution (use your sampling distribution from Problem 3 to answer Problems 4 and 5).

4. **Method 1: Counting Dots**

   (a) Based on counting dots in your sampling distribution, the **middle 95%** of the sample mean test scores land between ___________ points and ___________ points. Explain how you came up with these two values.

   Sample Response:
   The sample below is based on the sampling distribution presented in Problem 3(a).

   - To capture the middle 95% of the sample means I need to exclude 5% of the sample means. This means that I need to exclude 2.5% of the sample means from each tail end. Since we used 500 random samples of size 10 and 2.5% of 500 is 12.5, I need to exclude 12–13 dots from each tail end. When I do this, the middle 95% of the sample means land between 73.3 and 80.9.

   (b) The two values you found above in Problem 4(a) are both approximately ___________ points from the mean of your sampling distribution. Explain how you came up with your answer, which is a margin of error with a confidence level of 95%.

   Sample Response:
   Continuing from the sample response presented above in 4(a), we have that $76.9 - 73.3 = 3.6$ and $80.9 - 76.9 = 4$. These values are both about 3.8 points from the mean of the sampling distribution.

Commentary:
This method asks undergraduates to "count dots" (a technique they may be unfamiliar with) to determine

where the middle (in this case) 95% of the sample means land. Under the assumption that a sampling distribution is unimodal and symmetric, undergraduates could count the middle 95% of the dots on the sampling distribution, but a more efficient method would be to exclude 5% of the dots, or in other words, exclude 2.5% of the dots from either tail of the sampling distribution. Make sure both approaches are discussed in class.

Problem 5 prompts undergraduates to use the empirical rule to estimate a margin of error. From our experience, most undergraduates remember the empirical rule from high school, but remind them of the rule as needed. We have found that many undergraduates will recognize that they need to "go out" 2 standard deviations in the sampling distribution (i.e., 2 standard errors).

Class Activity Problem 5

5. **Method 2: Empirical Rule**
A student, Kyle, used the empirical rule instead of counting dots to estimate a margin of error with a confidence level of 95%. They said that using the empirical rule was quicker than counting dots and that they got a similar answer to their friends who counted dots.

(a) Use the empirical rule and your sampling distribution to estimate a margin of error with a confidence level of 95% (show your work). In other words, how many test score points do you need to go out from the mean of the sampling distribution to capture the **middle 95%** of the sample mean test scores? Compare your answer to your answer in Problem 4(b).

Sample Response:
The sample below continues to be based on the sampling distribution presented in Problem 3(a) as a sample response.

- The empirical rule tells me that the middle 95% of the data in the sampling distribution falls within 2 standard errors of the mean. So, we need to go out $2 \times 1.884 = 3.768$ points from the mean of 76.937 points.
- In 4(b), the margin of error was about 3.8 points which is very similar to 3.768 points found in 5(a)!

(b) Describe why Kyle might have thought to use the empirical rule to compute a margin of error in this situation.

Sample Response:
Kyle probably thought to use the empirical rule because they noticed that the sampling distribution is unimodal and symmetric and because we were looking to find the middle 95% of the sampling distribution.

(c) Explain why it's appropriate for Kyle to use the empirical rule to compute a margin of error in this situation.

Sample Response:
The empirical rule assumes that the distribution is unimodal and symmetric. Because our sampling distribution is approximately unimodal and symmetric, it is appropriate to use the empirical rule.

Commentary:
This method asks undergraduates to apply the empirical rule to determine where the middle 95% of the sample means land. We want undergraduates to recognize that this method is appropriate because (1) the sampling distribution is unimodal and symmetric and (2) we are looking for the middle 95% which directly ties to one of the three values (i.e., 68%–95%–99.7%) stated in the empirical rule.

Discuss This Connection to Teaching

The empirical rule is often taught in a high school algebra course. Students are expected to "use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages" and "recognize that there are data sets for which such a procedure is not appropriate" (CCSSM, 2010, p. 81). Relating the empirical rule to a margin of error associated with a 95% confidence level is a way prospective teachers can help their students understand that statistical methods are only as strong as the assumptions upon which they are built, and critical assessment of those assumptions is one of the most important steps of a statistical analysis.

Problem 6 focuses undergraduates' attention on responding to others' statistical conjectures, which you can emphasize by discussing the following connection to teaching. Further, this problem helps undergraduates recognize when the "counting dots" method may be more efficient than the empirical rule to estimate a margin of error, assuming that a sampling distribution is approximately unimodal and symmetric.

Discuss This Connection to Teaching

Problem 6 focuses on analyzing other students' thinking in order to develop undergraduates' skills in understanding school student thinking and guiding school students' understanding. It is valuable for all undergraduates (especially prospective teachers) to think about how others use, reason with, and communicate statistics. These problems also give prospective teachers (and tutors and future graduate students) an opportunity to think about how they would respond to student work in ways that nurture students' assets and understanding, and in ways that help develop students' statistical understanding.

Class Activity Problem 6

6. Luis is another student in the same class as Kyle. He noticed what Kyle did and wonders if it will always work. He asks the teacher if they can always use the empirical rule in this situation to estimate a margin of error. The teacher recognizes that this is an opportunity to help her students understand when these methods can be used to estimate a margin of error and when one method is more convenient over the other. The teacher ask her students the following question:

   If you are given a sampling distribution that is approximately unimodal and symmetric, describe how you could estimate a margin of error with a confidence level of 68% and another margin of error with a confidence level of 90%.

   (a) Explain how this question can help Luis to understand more about under what conditions the empirical rule is a convenient method for estimating a margin of error from a unimodal and symmetric sampling distribution.

   Sample Response:
   This question helps Luis see that there are certain middle percentages of a unimodal and symmetric distribution that can be quickly described by the empirical rule, and there are others that are not so easy. The empirical rule is quick and easy for margin of errors with a confidence level of 68%, 95% and 99.7%, which correspond to the mean of the sampling distribution +/- 1, 2, 3 SD of the sampling distribution (i.e., the SE of the sample statistic), respectively. So, the 68% prompt connects to margin of error to 1 SD, but the 90% prompt doesn't have an exact match from the empirical rule.

(b) What other confidence levels could the teacher ask about to help students understand under what conditions the empirical rule is (or is not) a convenient method for estimating a margin of error from a unimodal and symmetric sampling distribution? Explain.

Sample Response:
The teacher could ask about the other two values in the empirical rule. That is, 95% and 99.7% to demonstrate when the empirical rule is a convenient method for estimating a margin of error. Any other percent besides 68%, 95% and 99.7% will likely be more convenient to use the counting dots method.

Class Activity: Problem 7 (5 minutes)

Pass out **Problem 7** of the Class Activity, which asks undergraduates to return to the context presented at the beginning of the Class Activity (investigating test scores for seniors at Maplewood High School) and write a sentence that summarizes what they discovered. The goal is for undergraduates to use their estimate of the population parameter (i.e., their sample mean from Problem 1(a)) and a margin of error (constructed using their approximate sampling distribution) to write a sentence similar to the following from the Pre-Activity: *"A local TV weather forecaster estimates that the region's average winter snowfall is 23 inches, with a margin of error of* $\pm$ *2 inches."*

Class Activity Problem 7

7. Use the mean from your first random sample of 10 test scores (see Problem 1(a)) and the margin of error you found using your approximate sampling distribution (see Problem 4 or 5) to write a sentence that describes what you found about the typical test score for seniors at Maplewood High School. Your sentence can be modeled after the sentence describing snowfall in Problem 1 of the Pre-Activity.

Sample Response:
We estimate that the average test score for seniors at Maplewood High School is 76.1 points $\pm$ 3.8 points.

Commentary:

- Make sure undergraduates are using their sample mean from Problem 1(a) and not the center of their approxiamte sampling distribution as their point estimate. You can remind them that in the real-world, we often get only one sample. If that sample is a random sample, then the sample mean is our best guess at the true population mean.
- We have seen some undergraduates chose to write their answer in terms of a confidence interval (e.g., $(72.3, 79.9)$ points). If this occurs, then optionally and as appropriate for your class, focus on guiding undergraduates to correctly interpret their interval in the context of the problem. For example, you can have undergraduates compare their range of plausible values to the true mean. Then, undergraduates may notice that some of their peers' "intervals" contained the true mean and some did not.
- Alternatively, if undergraduates do not write their answer in terms of a confidence interval, this problem may be extended to introduce the concept of a confidence interval, and we leave it up to the instructor whether they want to spend time in this lesson or subsequent lessons defining and interpreting a confidence interval.

Wrap-Up (5 minutes)

Conclude the lesson by briefly discussing under what conditions a sample statistic is a good estimate of a population parameter, how undergraduates used simulation to construct a sampling distribution and build an understanding of margin of error, and why it is valuable to report a margin of error. This discussion may include the following prompts and ideas:

- In the Class Activity, how did the center of the approximate sampling distribution compare to the center of the population distribution? Do you think this will always be the case? What implications does this have in the real-world, where we often do not have access to the population distribution and can only rely on one random sample?
  – In the Class Activity, we saw (on average) how the sample means are really close to (if not exactly at) the population mean. This allows us to use a single sample mean (from a random sample) as our best guess at the true population mean. In other words, the sample mean is an unbiased estimator of the typically unknown population mean.
- Why is it valuable to report a margin of error?
  – A margin of error is valuable to report when estimating a population parameter because although a sample statistic may be a good guess at the population parameter, we know that there is inherent variability in the sampling strategy used. If we took another sample of the same size our "best guess" would change slightly (as we saw in the Class Activity). A margin of error lets us represent a range of plausible values we could expect to observe from a single sample of the population of interest.
- How did we use simulation to develop an understanding of margin of error?
  – Review how undergraduates set up a simulation to generate an approximate sampling distribution. Then, they used their sampling distribution and either counted dots or used the empirical rule to estimate a margin of error with a 95% confidence level.

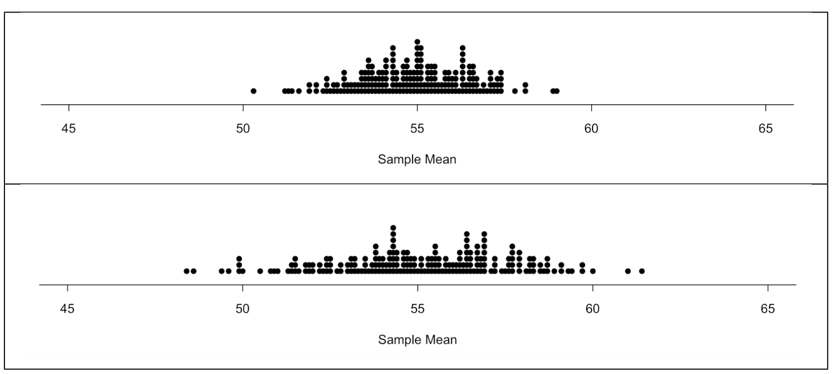
Homework Problems

At the end of the lesson, assign the following homework problems, and assign any additional homework problems at your discretion.

Problem 1 presents two sampling distributions of sample means, and asks undergraduates to explain what the population mean is and to estimate a margin of error for a confidence level of 95%. Because the standard error is not given, undergraduates need to rely on the "counting dots" method to construct a margin of error.

Homework Problem 1

1. Consider the two simulated sampling distributions of sample means.



   (a) Assume the sample means came from a random sample. What's your best guess at where the population mean is? Explain.

   Sample Response:

   - A good estimate of the population mean is about 55 because both sampling distributions are centered around 55, and if we have used a random sample, then the center of a sampling distribution of sample means will be close to the true population mean.

(b) Based on the top sampling distribution, what is a reasonable estimate of a margin of error for a confidence level of 95%? Explain your reasoning. (Note that the sampling distribution was created using 200 samples of size 40.)

Sample Response:

I need to capture the middle 95% of the sample means which means that I need to exclude 2.5% of the sample means from each tail end. Since the sampling distribution was created with 200 random samples of size 40 and 2.5% of 200 is 5, I need to exclude 5 dots from each tail end. When I do this, the middle 95% of the sample means land between 49.9 and 59.4. The mean of the sampling distribution is about 55. Since $55 - 49.9 = 5.1$ and $59.4 - 55 = 4.4$, a reasonable estimate of the margin of error for a confidence level of 95% is about 4.8.

In Problem 2, undergraduates consider why a margin of error is reported and discuss what understanding students do and do not yet have when they select an incorrect answer. This problem is adapted from a problem created by the *LOCUS Project* (Levels of Conceptual Understanding in Statistics). You can view commentary and correct answers to the original problem at `https://locus.statisticseducation.org/professional-development/questions/interpret-results?type=prodev_multiple_choice_question&field_prodev_level_tid=8`.

The LOCUS Project is a useful resource for future teachers to be aware of as they can potentially use these kinds of questions in their future classrooms. The problems from the LOCUS Project have been developed to measure students' understanding across levels of development (elementary, middle school, high school) as identified in the Pre-K–12 Guidelines for Assessment and Instruction in Statistics Education II (GAISE II) Report (Bargagliotti et al., 2020), and they align with the Common Core State Standards for Mathematics.

Homework Problem 2

2. Saskia, Aaron, Gerlie, and Moses are working on the following problem.

> A survey of 625 randomly selected students was conducted to determine the average amount of time students sleep during a weekday. The survey reported an average of 6.5 hours. The survey estimate had a margin of error of half an hour. A margin of error is reported because
>
> A. Sample means vary from sample to sample.
>
> B. Students may intentionally respond incorrectly.
>
> C. Students may misunderstand the survey questions.
>
> D. The people doing the survey may have recorded results incorrectly.

Each student selects a different reason for why a margin of error is reported.

Choice A	**Choice B**	**Choice C**	**Choice D**
Gerlie	Saskia	Aaron	Moses

(a) Who selected the correct (and complete) answer and why?

Sample Undergraduate Responses:

- The correct answer is A because random samples vary which mean the sample means will also vary.
- Gerlie is the most correct because two simple random samples will most likely be different, even from the same population.
- Gerlie is correct because variability is inherent in sampling methods.

(b) For each of the three students who selected an incorrect choice, explain what conception they have of margin of error.

Sample Undergraduate Responses:

- Moses likely has some understanding but doesn't understand the point that sample variation is not necessarily a result of collection error.
- The three other students may be taking margin of error too literally. They may see the word error and assume that the data given were wrong or it was interpreted wrong, when it really means that the average that was determined for that sample could vary if the survey was replicated with a different 625 randomly selected students.
- Each response is similar in that they all have to do with the surveyed students themselves and not the data. They all sound like an error that could occur.
- Saskia, Aaron, and Moses may think that "error" means someone is at fault—whether it be the people giving the survey or the students taking the survey.

Problem 3 asks undergraduates to consider (and explain) what will happen to a margin of error when you increase the confidence level.

Homework Problem 3

3. Recall the context of test scores from the Class Activity. Suppose that you wanted to capture the middle 99% (instead of the middle 95%) of the mean test scores from a sample of size 10. How would your margin of error change? Explain your reasoning.

Sample Responses:

- Because we are capturing more of the sample mean test scores, we need to go out further from the mean of the sampling distribution which will make the margin of error larger.
- The ME will increase because we are getting a larger scope of values to see.

Problem 4 prompts undergraduates to describe how they can create a range of plausible values. Undergraduates will describe how they can use a random sample to estimate an unknown value of a population and also describe how to conduct a simulation to construct a sampling distribution and estimate a margin of error. This problem is modeled after the "Gettysburg Address Problem" (see Chance & Rossman, 2006).

Homework Problem 4

4. The proliferation of text generated by artificial intelligence has led to questions about how to distinguish passages that are written by humans compared to passages written by artificial intelligence, which leads to the need to examine characteristics of blocks of text. One characteristic of a block of text is the mean word length. Consider the excerpt of Martin Luther King Jr.'s "I Have a Dream" speech.

> And so even though we face the difficulties of today and tomorrow, I still have a dream. It is a dream deeply rooted in the American dream. I have a dream that one day this nation will rise up and live out the true meaning of its creed: We hold these truths to be self-evident, that all men are created equal.
>
> I have a dream that one day on the red hills of Georgia, the sons of former slaves and the sons of former slave owners will be able to sit down together at the table of brotherhood.
>
> I have a dream that one day even the state of Mississippi, a state sweltering with the heat of injustice, sweltering with the heat of oppression, will be transformed into an oasis of freedom and justice.
>
> I have a dream that my four little children will one day live in a nation where they will not be judged by the color of their skin but by the content of their character. I have a dream today.

Draw on your experiences from the Class Activity to estimate the mean word length of this passage with some margin of error:

(a) Without going through and calculating the length of every single word in the passage, describe how you would use statistics to construct a "best guess" at the true mean word length.

Sample Response:
I would randomly select 10 words, count how many letters are in each word, and compute the mean. This would give me a sample mean word length and since I used a random sample, it will be my best guess at the population mean word length.

(b) Describe how you would use simulation to construct a margin of error for the true mean word length?

Sample Response:
I would randomly select 10 words from the passage, record their length, and then compute the mean length from my sample. I would then repeat this process for a total of 500 times. Now I have 500 sample means. I can create a dotplot of these sample means and this would represent my approximate sampling distribution. I would use the mean of my first random sample as an estimate of the population mean word length. To create a plausible range of values for the mean word length, I would first count where the middle 95% of the dots in the sampling distribution are and see how far each is from the mean of the sampling distribution. This would give me a margin of error with a 95% confidence level. Finally, my range of plausible values would be constructed by computing "sample mean $\pm$ ME".

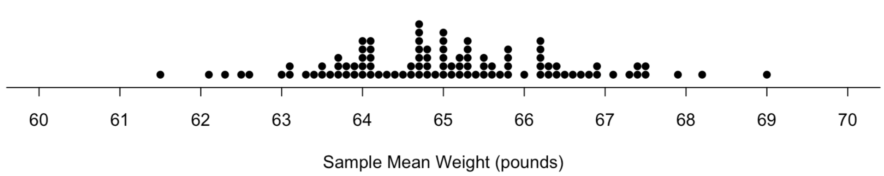
Assessment Problems

The following two problems address ideas explored in the lesson, with a focus on connections to teaching and mathematical content. You can include these problems as part of your usual course quizzes or exams.

Problem 1 assess undergraduates' understanding of how a dotplot was created, how to develop a margin of error given a sampling distribution, and what happens to a margin of error when the associated confidence level is increased.

Assessment Problem 1

1. Below is a dotplot of the sample mean weight for 100 different random samples of size 10 from a population of adult Labrador retrievers where the mean weight is 65 pounds.



(a) Describe what one dot in the dotplot represents.

Sample Responses:

- The mean weight (in pounds) of a random sample of 10 retrievers.
- 10 adult Labrador retrievers were randomly selected and their body weight was recorded. One dot represents the mean of these 10 body weights.

(b) Fill in the blanks.

95% of the sample mean weights fall between ________________ *and* ________________.

Explain how you came up with these endpoints.

Sample Undergraduate Responses:

- 95% of the sample mean weights fall between 62.4 pounds and 67.8 pounds. To figure out where the middle 95% of the sample means were, I noticed that there are 100 samples shown in the sampling distribution. Therefore, I needed to capture all but 5 of the dots (or about 2.5 dots on each end).
- 62.3 and 67.7. I came up with these points by "chopping" off 2.5% of the dots on either side.
- 62.4 and 67.8. I found that 95% of 100 is 95, so I needed to count in 2.5 dots on each side to remove the other 5%, so the number of dots between my two endpoints marks 95% of the dots.

(c) Based on your answer in 1(b), estimate a margin of error with a confidence level of 95%. Explain your work.

Sample Responses:

Answers will vary depending on the endpoints undergraduates found in 1(b). The sample responses below correspond to the three sample responses in 1(b).

- The population mean is 65 pounds and the interval above should be approximately $65 \pm \text{ME}$. Since $65 - 62.4 = 2.6$ and $67.8 - 65 = 2.8$, the margin of error is about 2.7 pounds.
- $62.3 + 2.7 = 65$ and $67.7 - 2.7 = 65$. So the $\text{MoE} = 2.7$ pounds.
- I took the endpoints above, found the distance between the two values and divided by 2 to find the margin of error. $67.8 - 62.4 = 5.4$ and $5.4 \div 2 = 2.7$

(d) Would a margin of error with a confidence level of 99% be larger or smaller than the margin of error you estimated in 1(c)? Explain your reasoning.

Sample Responses:

- A margin of error with a confidence level of 99% would be larger than a margin of error with a confidence level of 95% because we are capturing more of the sample means, so the two endpoints would be farther from the mean.
- With a 99% confidence level we are capturing more sample means. This means the margin of error will be larger.

Problem 2 assesses undergraduates' understanding of the two methods they used in the Class Activity to estimate a margin of error, given a sampling distribution. In this problem, they will explain when one method is more efficient over the other, under the assumption that a sampling distribution is unimodal and symmetric.

Assessment Problem 2

2. Kyle's and Luis' teacher knows that her students can count dots and use the empirical rule on a unimodal and symmetric sampling distribution to estimate a margin of error. It's because the sampling distribution is symmetric and unimodal that both methods will give approximately the same answer for a margin of error. The teacher wants her students to understand when each method is (or is not) the most useful method for estimating a margin of error in this situation.

(a) The teacher gives her students a unimodal and symmetric distribution and asks them to estimate a margin of error with a confidence level of 68%. Explain why the teacher uses this prompt to help her students understand when the empirical rule is more useful than the counting dots method to estimate a margin of error associated with a 68% confidence level.

Sample Response:
The teacher uses a confidence level of 68% because it's one of the three values in the empirical rule. All students would need to do is to compute 1 standard deviation of the sampling distribution. This is much quicker than counting the middle 68% of the dots in the sampling distribution.

(b) The teacher then asks her students to use the same sampling distribution from part (a) to now estimate a margin of error with a confidence level of 90%. Explain why this question is useful in helping students understand when the counting dots method is more useful than the empirical rule to estimate a margin of error associated with a 90% confidence level.

Sample Response:
Because 90% is not one of the three numbers in the empirical rule, it would not be a single calculation (like 3 times the standard deviation of the sampling distribution) to estimate a margin of error. Students would need to figure out how to get 90% from using 68%, 95%, and 99.7% to use the empirical rule. At this point, it would be more efficient to count the middle 90% of the sample means, or rather exclude 5% of the sample means from each tail in the sampling distribution. Then find the distance between those endpoints and the mean of the sampling distribution to construct a margin of error associated with a 90% confidence level.

1.6 References

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[2] Burrill, G. (2021). Simulation as a tool to make sense of the world. In R. Helenius & E. Falck (Eds.), *Statistics education in the era of data science: Proceedings of the satellite conference of the International Association for Statistical Education (IASE).*

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[4] De Veaux, R. D., Velleman, P. F., & Bock, D. E. (2012). *Stats: Data and models* (3rd ed.). Pearson Education.

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[6] National Governors Association Center for Best Practices & Council of Chief State School Officers (2010). *Common Core State Standards for Mathematics.* Authors. Retrieved from `http://www.corestandards.org/`

[7] Peck, R., Gould, R., Miller, S., Wilson, P., & Zbiek, R. (2013). *Developing essential understanding of statistics for teaching mathematics in grades 9–12*. National Council of Teachers of Mathematics.

[8] StatKey Simulation Environment. (Accessed April 7, 2023). `http://www.lock5stat.com/StatKey/`.

1.7 Lesson Handouts

Handouts for use during instruction are included on the pages that follow. LaTeX files for these handouts can be downloaded from INSERT URL HERE.

NAME: **PRE-ACTIVITY:** UNDERSTANDING MARGIN OF ERROR (page 1 of 1)

1. Consider the following sentence from a statistical report, as presented in De Veaux, Velleman, and Bock (2012).

   Based on meteorological data for the past century, a local TV weather forecaster estimates that the region's average winter snowfall is 23 inches, with a margin of error of $\pm$ *2 inches.*

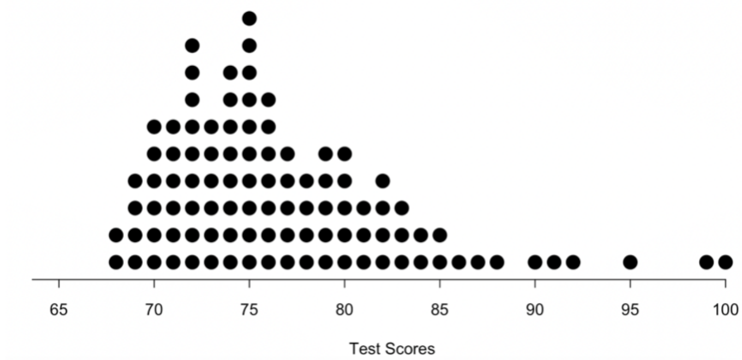
   (a) If you lived in this region, would you want the margin of error to be large or small? Explain.

   (b) Why do you think a margin of error is reported?

NAME: **CLASS ACTIVITY:** UNDERSTANDING MARGIN OF ERROR (page 1 of 5)

Investigating Test Scores for Seniors at Maplewood High School

Suppose that our entire population is 100 student test scores taken from the seniors at Maplewood High School (see the *population distribution* below), and we know the population mean test score (out of 100 points) is $\mu = 76.96$ points.



1. You have a set of 100 cards that represent the 100 test scores from the entire population of seniors at Maplewood High School. Randomly draw a sample of **size 10** from these cards.

   (a) Write down the 10 test scores you randomly selected and compute the mean test score of your sample.

   (b) Share your sample mean test score with classmates and compare. Why do you think your sample mean test score is different from others?

2. What happens if we repeat this process? From your set of 100 cards, continue to randomly draw a sample of size 10 and compute the mean test score of your sample. Repeat this process until you have a total of 10 sample means.

   (a) Write the 10 sample mean test scores below.

   (b) As a class, create a dotplot of everyone's sample means and sketch it below. This is the class's *approximate sampling distribution* of sample mean test scores. Describe what one dot in the dotplot represents.

We can continue to perform a simulation by hand with the cards, but technology provides us a more efficient way to do this! Our goal is to see what other sample means we might have gotten from different samples of size 10, and simulation is a tool we can use to see how variable the sample mean test scores might be.

3. Using the population of 100 test scores, use technology to conduct a simulation. Randomly draw a sample of **size 10** and compute the mean test score of your sample. Repeat this process for a total of 500 times.

   (a) Create a dotplot of your 500 sample mean test scores (i.e., another approximate sampling distribution) and sketch it below. Describe what one dot in the dotplot represents.

   (b) Describe the shape, center, and spread of your sampling distribution from Problem 3(a). What is the mean and standard deviation of your sampling distribution? Note that the standard deviation of a sampling distribution is referred to as a *standard error*.

   (c) Compare the shape, center, and spread of your sampling distribution from Problem 3(a) to the population distribution presented on page 1 of the Class Activity.

   (d) Based on your sampling distribution from Problem 3(a), is a sample mean (computed from a random sample of size 10) a good way to estimate the population mean test score? Explain your reasoning.

In Problem 3, we found that a sample mean (computed from a random sample) is usually a really good guess of the population mean. But, not all good guesses are created equally! We also saw in that sampling distribution that some of our good guesses were further from the population mean than others. In statistics, we care about how much variability is present among these sample means (estimated by the standard error you found in Problem 3), and we often report a range of plausible values for a population mean. Problems 4 and 5 guide you through two methods for estimating a margin of error from a unimodal and symmetric sampling distribution (use your sampling distribution from Problem 3 to answer Problems 4 and 5).

4. **Method 1: Counting Dots**

   (a) Based on counting dots in your sampling distribution, the **middle 95%** of the sample mean test scores land between ___________ points and ___________ points. Explain how you came up with these two values.

   (b) The two values you found above in Problem 4(a) are both approximately ___________ points from the mean of your sampling distribution. Explain how you came up with your answer, which is a margin of error with a confidence level of 95%.

5. **Method 2: Empirical Rule**
   A student, Kyle, used the empirical rule instead of counting dots to estimate a margin of error with a confidence level of 95%. They said that using the empirical rule was quicker than counting dots and that they got a similar answer to their friends who counted dots.

   (a) Use the empirical rule and your sampling distribution to estimate a margin of error with a confidence level of 95% (show your work). In other words, how many test score points do you need to go out from the mean of the sampling distribution to capture the **middle 95%** of the sample mean test scores? Compare your answer to your answer in Problem 4(b).

(b) Describe why Kyle might have thought to use the empirical rule to compute a margin of error in this situation.

(c) Explain why it's appropriate for Kyle to use the empirical rule to compute a margin of error in this situation.

6. Luis is another student in the same class as Kyle. He noticed what Kyle did and wonders if it will always work. He asks the teacher if they can always use the empirical rule in this situation to estimate a margin of error. The teacher recognizes that this is an opportunity to help her students understand when these methods can be used to estimate a margin of error and when one method is more convenient over the other. The teacher ask her students the following question:

   If you are given a sampling distribution that is approximately unimodal and symmetric, describe how you could estimate a margin of error with a confidence level of 68% and another margin of error with a confidence level of 90%.

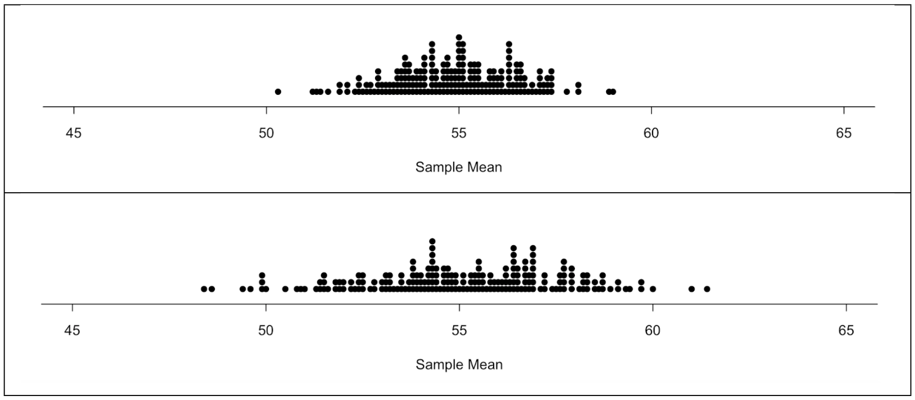
   (a) Explain how this question can help Luis to understand more about under what conditions the empirical rule is a convenient method for estimating a margin of error from a unimodal and symmetric sampling distribution.

   (b) What other confidence levels could the teacher ask about to help students understand under what conditions the empirical rule is (or is not) a convenient method for estimating a margin of error from a unimodal and symmetric sampling distribution? Explain.

7. Use the mean from your first random sample of 10 test scores (see Problem 1(a)) and the margin of error you found using your approximate sampling distribution (see Problem 4 or 5) to write a sentence that describes what you found about the typical test score for seniors at Maplewood High School. Your sentence can be modeled after the sentence describing snowfall in Problem 1 of the Pre-Activity.

NAME: **HOMEWORK PROBLEMS:** UNDERSTANDING MARGIN OF ERROR (page 1 of 2)

1. Consider the two simulated sampling distributions of sample means.



   (a) Assume the sample means came from a random sample. What's your best guess at where the population mean is? Explain.
   (b) Based on the top sampling distribution, what is a reasonable estimate of a margin of error for a confidence level of 95%? Explain your reasoning. (Note that the sampling distribution was created using 200 samples of size 40.)

2. Saskia, Aaron, Gerlie, and Moses are working on the following problem.

> A survey of 625 randomly selected students was conducted to determine the average amount of time students sleep during a weekday. The survey reported an average of 6.5 hours. The survey estimate had a margin of error of half an hour. A margin of error is reported because
>
> A. Sample means vary from sample to sample.
> B. Students may intentionally respond incorrectly.
> C. Students may misunderstand the survey questions.
> D. The people doing the survey may have recorded results incorrectly.

Each student selects a different reason for why a margin of error is reported.

Choice A	**Choice B**	**Choice C**	**Choice D**
Gerlie	Saskia	Aaron	Moses

   (a) Who selected the correct (and complete) answer and why?
   (b) For each of the three students who selected an incorrect choice, explain what conception they have of margin of error.

3. Recall the context of test scores from the Class Activity. Suppose that you wanted to capture the middle 99% (instead of the middle 95%) of the mean test scores from a sample of size 10. How would your margin of error change? Explain your reasoning.

4. The proliferation of text generated by artificial intelligence has led to questions about how to distinguish passages that are written by humans compared to passages written by artificial intelligence, which leads to the need to examine characteristics of blocks of text. One characteristic of a block of text is the mean word length. Consider the excerpt of Martin Luther King Jr.'s "I Have a Dream" speech.

   > And so even though we face the difficulties of today and tomorrow, I still have a dream. It is a dream deeply rooted in the American dream. I have a dream that one day this nation will rise up and live out the true meaning of its creed: We hold these truths to be self-evident, that all men are created equal.
   >
   > I have a dream that one day on the red hills of Georgia, the sons of former slaves and the sons of former slave owners will be able to sit down together at the table of brotherhood.
   >
   > I have a dream that one day even the state of Mississippi, a state sweltering with the heat of injustice, sweltering with the heat of oppression, will be transformed into an oasis of freedom and justice.
   >
   > I have a dream that my four little children will one day live in a nation where they will not be judged by the color of their skin but by the content of their character. I have a dream today.

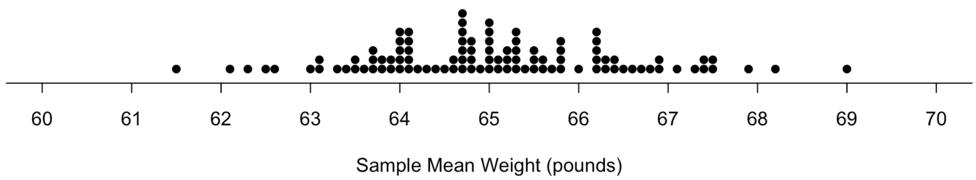
   Draw on your experiences from the Class Activity to estimate the mean word length of this passage with some margin of error:

   (a) Without going through and calculating the length of every single word in the passage, describe how you would use statistics to construct a "best guess" at the true mean word length.

   (b) Describe how you would use simulation to construct a margin of error for the true mean word length?

NAME: **ASSESSMENT PROBLEMS:** UNDERSTANDING MARGIN OF ERROR (page 1 of 2)

1. Below is a dotplot of the sample mean weight for 100 different random samples of size 10 from a population of adult Labrador retrievers where the mean weight is 65 pounds.



(a) Describe what one dot in the dotplot represents.

(b) Fill in the blanks.

95% of the sample mean weights fall between ________________ *and* ________________.

Explain how you came up with these endpoints.

(c) Based on your answer in 1(b), estimate a margin of error with a confidence level of 95%. Explain your work.

(d) Would a margin of error with a confidence level of 99% be larger or smaller than the margin of error you estimated in 1(c)? Explain your reasoning.

ASSESSMENT PROBLEMS: UNDERSTANDING MARGIN OF ERROR (page 2 of 2)

2. Kyle's and Luis' teacher knows that her students can count dots and use the empirical rule on a unimodal and symmetric sampling distribution to estimate a margin of error. It's because the sampling distribution is symmetric and unimodal that both methods will give approximately the same answer for a margin of error. The teacher wants her students to understand when each method is (or is not) the most useful method for estimating a margin of error in this situation.

   (a) The teacher gives her students a unimodal and symmetric distribution and asks them to estimate a margin of error with a confidence level of 68%. Explain why the teacher uses this prompt to help her students understand when the empirical rule is more useful than the counting dots method to estimate a margin of error associated with a 68% confidence level.

   (b) The teacher then asks her students to use the same sampling distribution from part (a) to now estimate a margin of error with a confidence level of 90%. Explain why this question is useful in helping students understand when the counting dots method is more useful than the empirical rule to estimate a margin of error associated with a 90% confidence level.