Fall 2017 - M172 Review Session - Oct. 15/16, 2017 - EXAM II

1. Circle the appropriate choice of TRUE or FALSE.

TRUE FALSE: You could use $x = 4\sec\theta$ to evaluate $\int \frac{1}{\sqrt{x^2-4}}\,dx$

TRUE FALSE: Using partial fraction decomposition, $\frac{1}{(1+x)^2} = \frac{Ax+B}{(1+x)^2}$

TRUE FALSE: The formula for determining area of a surface of revolution about the x-axis is $S = 2\pi \int_a^b f(x)\sqrt{1+f'(x)}\,dx$

TRUE FALSE: $\int_0^5 \frac{1}{(x-3)^2}\,dx = \frac{-1}{x-3}\Big|_0^5 = \frac{-1}{2} - \frac{1}{3} = \frac{-5}{6}$

TRUE FALSE: If $2x = 3\sin\theta$, then $\sqrt{9-4x^2} = 3\cos\theta$

TRIG INTEGRALS / TRIG SUB

2. Evaluate each integral using trigonometric identities where appropriate

(a) $\int \tan^3 x \sec^3 x\,dx$

(b) $\int \cos^3 x\,dx$

(c) $\int \tan^2 x\,dx$

(d) $\int \sin 8x \sin 5x\,dx$

(e) $\int_0^\pi (1+\sin x)^2\,dx$

(f) $\int \cos\sqrt{x}\,dx$

3. Evaluate each integral using trigonometric substitution where appropriate

(a) $\int x^3\sqrt{1-x^2}\,dx$

(b) $\int \frac{1}{\sqrt{x^2+16}}\,dx$

(c) $\int \frac{x^3}{\sqrt{x^2+100}}\,dx$

(d) $\int \frac{1}{x^3\sqrt{x^2-2}}\,dx$

(e) $\int \frac{1}{(4+9x^2)^2}\,dx$

(f) $\int \frac{x+\arcsin x}{\sqrt{1-x^2}}\,dx$

PARTIAL FRACTIONS

4. Evaluate each integral using the methods of partial fraction decomposition where appropriate

(a) $\int \frac{1}{x^2-1}\,dx$

(b) $\int \frac{x^3-2x^2-4}{x^3-2x^2}\,dx$

(c) $\int \frac{x+9}{x(x^2+9)}\,dx$

(d) $\int \frac{1}{x^2+4x+5}\,dx$

(e) $\int \frac{\sqrt{x}}{x-4}\,dx$

(f) $\int \sqrt{1+e^x}\,dx$

IMPROPER INTEGRALS

5. Determine whether the improper integral converges, and if so evaluate it.

(a) $\int_{-\infty}^{-1} \frac{1}{\sqrt{2-x}}\,dx$

(b) $\int_1^\infty \frac{1}{(3x+1)^2}dx$

(c) $\int_0^\infty \frac{\arctan x}{1+x^2}\,dx$

(d) $\int_{-1}^0 \frac{3x}{(x+1)(x-2)}dx$

(e) $\int_0^\infty xe^{-x}\,dx$

(f) $\int_0^2 x^2\ln x\,dx$

COMPARISON THEOREM

6. Use the Comparison Theorem to determine whether or not the integral converges.

(a) $\int_1^\infty \frac{1}{x^9+2}\,dx$ (b) $\int_5^\infty \frac{x}{\sqrt{x^4-1}}\,dx$ (c) $\int_0^1 \frac{1}{x+x^{1/3}}\,dx$ (d) $\int_0^{\pi/2} \frac{\cos x}{\sqrt{x}}\,dx$

APPLICATIONS

7. Find a constant C such that $p(x)$ is a probability density function on the given interval, and compute the probability indicated.

(a) $p(x) = Cx^2$ on $[0,3]$; $P(1 \le X \le 2)$

(b) $p(x) = \frac{C}{x^2+1}$ on $[0,\infty)$; $P(X \ge 1)$

(c) $p(x) = \frac{C}{\sqrt{4-x^2}}$ on $[0,2)$; $P(0 \le X \le 1)$

8. Calculate the arc length of the function over the given interval.

(a) $y = 3 + 4x^{3/2}$, $[0,1]$

(b) $y = \ln(\sec x)$, $[0,\pi/4]$

(c) $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x$, $[1,2e]$

9. Compute the surface area of revolution about the x-axis over the given interval.

(a) $y = x^3$, $[0,1]$

(b) $y = \sqrt{1+4x}$, $[1,5]$

(c) $y = \frac{2}{3}x^{3/2} - \frac{1}{2}x^{1/2}$, $[1,2]$

10. A plate is designed in the shape of the region under $y = \sin x$ for $0 \le x \le \frac{\pi}{2}$. **Set up but do not integrate** an integral to find the fluid force on the plate if it is submerged in 1 meter of a fluid of density 700 $\frac{kg}{m^3}$.

11. The end of a trough has the shape of a trapezoid with width 2 m at the bottom, width 4 m at the top, and height 2 m. See Figure 1 below. If the trough is filled to the top with orange juice with density $\rho = 1150\frac{kg}{m^3}$, find the fluid force on the trapezoidal end of the trough.

12. Calculate the fluid force on a semicircular plate of radius 1 m centered at the origin and oriented as shown in Figure 2. The surface of the fluid of density ρ is at $y = 2$.

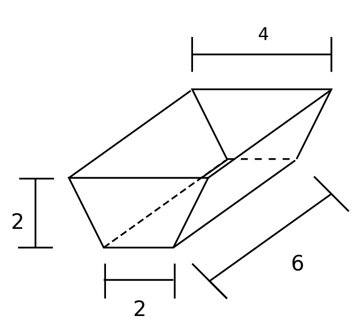

Figure 1: A trough.

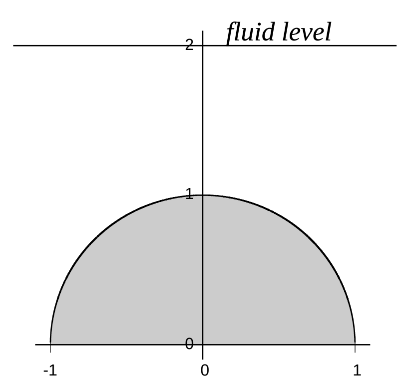

Figure 2: A plate.

The following information is given on the exam.

Indefinite integrals and Trigonometric Identities

Some trigonometric identities which may or may not be needed include:

$$\begin{aligned}
\cos^2 x &= \frac{1}{2}(1+\cos 2x) \\
\sin^2 x &= \frac{1}{2}(1-\cos 2x) \\
\sin 2x &= 2\ \sin x\ \cos x
\end{aligned}$$

$$\begin{aligned}
\sin A\ \cos B &= \frac{1}{2}(\sin(A+B)+\sin(A-B)) \\
\cos A\ \cos B &= \frac{1}{2}(\cos(A+B)+\cos(A-B)) \\
\sin A\ \sin B &= \frac{1}{2}(\cos(A-B)-\cos(A+B))
\end{aligned}$$

Some integrals which may or may not be needed include:

$$\begin{aligned}
\int \frac{du}{a^2+u^2} &= \frac{1}{a}\arctan\left(\frac{u}{a}\right)+c \\
\int \frac{du}{\sqrt{a^2-u^2}} &= \arcsin\left(\frac{u}{a}\right)+c \\
\int \sec u\ du &= \ln|\sec u+\tan u|+c \\
\int \sec^3 u\ du &= \frac{1}{2}\sec u\tan u+\frac{1}{2}\ln|\sec u+\tan u|+c \\
\int \csc u\ du &= -\ln|\csc u+\cot u|+c
\end{aligned}$$