# Solutions Manual

for Chapter 1

of

# PROOF:
## INTRODUCTION TO HIGHER MATHEMATICS

Seventh Edition

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**Note: Many problems do not have a unique correct answer**. The answer given here is among the best, but this is a language course, and languages permit different ways to say the same thing.

1) Different words may be used to express the same thought.
2) Some questions request the statement of a theorem. Different letters may express the same theorem. Theorems stated using variables $a$, $b$, and $c$ may be expressed using $x$, $y$ and $z$.
3) Some problems request pronunciations, and pronunciations may vary.
4) Some problems request counterexamples and many different counterexamples would work. This manual usually gives one counterexample, but not all possible counterexamples. The student's answer might not (probably should not!) duplicate the one in this manual.
5) Proofs are by no means unique. This manual gives one typical, efficient, proof, but other proofs are possible.

# Chapter 1: Introduction to Proofs

## Section 1.1. Preview of Proof

A1. a) False (it refers to the fifth major result). b) F c) T d) F
A3. a) F b) T c) T d) F
A5. Yes A7. No. ($x$ = -3)
A9. Yes A11. No.
^^^^^^^^
B1. [See Definition 2 of *placeholder*.]
B3. $R \subset S$ iff if $x \in R$, then $x \in S$.

^^^^ B5-14: Placeholders in B5,7,8,10,11,14 and not the others.
B5. P B7. P
B9. not P B11. P B13. not P
B15. a) A *generalization* is a sentence that asserts that something is always true. b) no.
c) An identity is an equation that is true for all values of the variable(s).
d) An identity can be regarded as an abbreviation of a true generalization, abbreviated to omit the "for all" quantifier.

B17. For all $b > 5$, $|b| > 5$ [or] For all $b$, if $b > 5$, then $|b| > 5$ .
B19. For all $a$ and $b$, $ab = 0$ iff $a = 0$ or $b = 0$.

B21. F. $x$ = -10. B23. F $x$ = -10 B25. T
B27. T B29. F $x$ = -3 B31. F $x$ = 1 and $c$ = ½.
B33. T ($x$ = ½) ) B35. F B37. T B39. F
B41. a) Yes. $x$ (such that the equation is true.) b) Yes. $b$, $c$, and $d$.
B43. [See Definition 1 of *proof*.] Logic and prior results.

## Section 1.2. Sets

A1. a) $\{x|3 < x < 8\}$ b) $\{x|x \le 5\}$ A3. a) $\{x|2 < x < 6.5\}$ b) $\{x|x \le 7.1\}$

A5. a) $(-\infty,6)$ b) $[-3, -1.5)$
A7. a) $[c,9)$ b) $(-\infty,2)$

A9. The set of (all) $x$ such that $x$ is less than two.
A11. The set of (all) $x$ such that $x$ is greater than or equal to 3.
A13. $x$ is in $S$. [or] $x$ is a member of $S$.
A15. $R$ is a subset of $S$.

A17. Two is in the set of (all) $x$ such that $x$ is less than or equal to three. (or, "is a member of")
A19. a) $(-7,12]$ b) $(-3, 5)$ c) $[0, 4)$
A21. a) $(-3, \infty)$ b) $(2, \infty)$
A23. a) $(-\infty,0]\cup[3,\infty)$ b) $(-\infty,2)\cup[4.1,\infty)$
A25. a) T b) T c) F
A27. a) F b) F c) F

A29. a) True b) False c) False
A31. a) False b) False c) False

A33. capital letters do not represent elements ($a \in S$ or $A \subset S$ would be ok)
A35. numbers are not subsets ($\{7\} \subset S$ or $7 \in S$ would be ok)
A37. ] should be )
A39. "and" connects sentences, not sets.
A41. "$\cap$" connects sets, not sentences.
A43. "$x \in S$" is ok, but "$\Rightarrow$" cannot connect to a set.
A45. "$\cup$" connects sets, not sentences.
A47. fine
A49. "or" connects sentences, not sets, and $(8,\infty)$ is a set.
A51. members of $T$ are represented by lower-case letters ($s \in T$ and $S \subset T$ would be ok)
A53. sets cannot equal numbers or sentences.
A55. members of $S$ are represented by lower-case letters ($a \in S$ and $A \subset S$ would be ok)
A57. "$\Rightarrow$" does not connect sets, and "$S \cup T$" is a set.
A59. "$\Rightarrow$" does not connect sets, and "$T$" is a set.

A61. fine
A63. $\infty$ has ), not ]
A65. $\infty$ is not a real number and cannot be in a set of real numbers
A67. $\{1,2,3,...,9\} = \{n \mid 0 < n < 10\} = \{n|1 \leq n \leq 9\}$.
A69. The usual picture shows $S$ completely inside and smaller than $T$, which might lead you to believe it must be smaller than $T$, which is not the case, according to the concept definition.

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B1. a) Figure 1.2.4 of *intersection* b) Figure 1.2.5 of *union* c) intersection
B3. Sketch something like Figures 9 for *intersection* and Figure 8 for *union*.

B7. a) "5" is a **number**. "{5}" is a **set** with one member, the number 5.
b) $5 \in \{5\}$. $5 \not\subset \{5\}$. $\{5\} \notin \{5\}$ [Use two of these.]
B9. [Sketch your own picture] with $a < b < c < d$. [Do not bother to label 0.]
a) $S \cap T = [b, c)$ b) $S \cup T = (a, d]$
c) $S^c = (-\infty, a] \cup [c, \infty)$ d) $R \cup T = [b, \infty)$
e) $T \subset S \cup R$
B11. and, intersection or, union not, complement

B13. $S^c = \{x \mid x \notin S\} = \{x \mid x$ is not a member of $S\}$

B15. a) $x \in S \cup T$. b) $x \in S$ or $x \in T$. c) iff.

B17. $x \in S \cap T$. $x \in S$ and $x \in T$. B19. $S \subset T$. If $x \in S$, then $x \in T$.
B21. It is false. Let $S = \{2, 3\}$ and $T = \{3, 4\}$. Then neither is a subset of the other.
B23. There are four: $\varnothing$, $\{2\}$, $\{5\}$, $\{2, 5\}$.
B25. a) The first and last are intervals of real numbers. The last includes $a$ and $b$ whereas the first does not. The second has only two elements.
b) $(a, b) \subset [a, b]$. $\{a, b\} \subset [a, b]$.
B27. a) It should read $S \cap T = \{7\}$. Intersections form sets, not numbers.
b) It should read $T \subset S$. The members of $S$ are numbers, not sets.
B29. -5
B31. It does not have one; 3 is not in it and for every element in it, there is a lesser one in it.

B33ff. [xx pic Insert Venn diagrams]
B39. a) F b) F c) T d) T

B41. $(-\infty,3) \cup (3,\infty)$.
B43. a) F b) T
B45. a) F b) T
B47. If $x \in S$, then $x \in T$.
B49. If $x \in R$, then $x \leq 36$.
B51. The second is always true, the second is not. $S \cap T \subset S$ is an abbreviated generalization.

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## Section 1.3. Logic for Mathematics

A1. a) expression b) expression c) sentence d) expression
A3. a) sentence b) expression c) expression d) expression
A5. b, f (the others are either not sentences or don't have variables)

A7. b, c A9. b A11. b
A13. $A \Rightarrow B$ A15. $(A \text{ or } B) \Rightarrow C$
A17. $A \Rightarrow B$ A19. $A \Rightarrow B$
A21. $A \Rightarrow B$ A23. $A \Rightarrow B$

A25. $H$ implies $C$ (or, "If $H$, then $C$.")
A27. $A$ and $B$ ]pause] implies $C$.
A29. a) $A \Rightarrow (B \text{ or } C)$ b) $(\text{not } A) \Rightarrow B$ c) $(A \text{ and } B) \Rightarrow C$.
A31. a) $[H \text{ and } (\text{not } C)] \Rightarrow D$ b) $(\text{not } B) \Rightarrow (\text{not } C)$ c) $B \Rightarrow (A \text{ or } C)$
A33. a) If $x > 7$, then $x > 5$. b) If $x \in T$, then $x \in$ S. c) If $B$, then $A$.
d) If $x \in S$, then $x \in T$. e) If $C$, then $D$.

A35 a) If $C$, then $B$. b) If $B$, then $C$. c) If $C$, then $B$.
A37. "$1 < x$ and $x < 5$."

A39. A sentence is a complete thought. An expression is a noun or a pronoun.
A41. 5 A43. 7
A45. Four rows cover all the possibilities of truth and falsehood: TT, TF, FT, FF

^^^^^^^^

B1. Your table should have 9 columns. **Partial** table.

| $H$ | $C$ | $H \Rightarrow C$ | not $C$ | not $H$ | (not $C$)$\Rightarrow$(not $H$) | (not $H$) or $C$ | $C\Rightarrow H$ | (not $H$)$\Rightarrow$(not $C$) |
|---|---|---|---|---|---|---|---|---|
| T | T | T | F | F | T | T | T | T |
| T | F | F | T | F | F | F | | |
| F | T | T | F | T | T | | | |
| F | F | T | T | | | | | |

f) $H \Rightarrow C$ is LE (not $C$)$\Rightarrow$(not $H$) is LE (not $H$) or $C$
$C\Rightarrow H$ is LE (not $H$)$\Rightarrow$(not $C$)

B3. a) Figures 2 and "and" and 3 for "or". b) See Figure 4, with TT being row 1, etc. c) "and"

B5. [Your choice. Illustrate *and*, *or*, and *not* as in Figures 6, 7, 5.]

B7& B9. **Partial** Table for B7 and B9

| $A$ | $B$ | $A \Rightarrow B$ | not $B$ | not $A$ | (not $B$)$\Rightarrow$(not $A$) | not($A\Rightarrow B$) |
|---|---|---|---|---|---|---|
| T | T | T | F | F | T | F |
| T | F | F | T | F | F | T |
| F | T | T | F | | | |
| F | F | T | T | | | |

Columns 3 and 6 show B7 is true.
Columns 6 and 9 show B9 is true.

B11. False. [Use a truth table.]
B13. $H \Rightarrow C$ is LE to (not $H$) or $C$. "or" can be evaluated one column at a time by concentrating on true components. When "not $H$" is T so is "(not $H$) or $C$" and when $C$ is T so is "(not $H$) or $C$." In the remaining row or rows $H \Rightarrow C$ is false.
B15. a) T,F or F,T b) T,T c) impossible

^^^^ B17-18. All could be misinterpreted. Order matters! All need parentheses, or at least a convention.
B17. a) T, T, F ($A$ or $B$) $\Rightarrow C$
b) T, F, F no convention. Use parentheses.
c) $A$ is F, $B$ is F (not $A$) and $B$ d) Use parentheses! (or, know the conventions)

B19. a) No, the associative law for "and" can be proved with an eight-line truth table.
b) Yes, "and" is associative.

B21. False. Knowing $C$ is T does not determine if $D$ is T.
B23. $x \le 5$ or $x > 3$.
B25. $x \notin S$ or $x \in T$.
B27. If the region where $H$ is T and $C$ is F is empty, then "$H \Rightarrow C$" is always true and so is "(not $H$) or $C$." If the figure looks like Figure 8, it is easy to see that "(not $H$) or $C$" must be true. The figure for "not $H$" shades all but the innermost region. "$C$" shades that innermost region and more. So, "(not $H$) or $C$" shades the entire box – it would always be true whenever "$H\Rightarrow C$" is.

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## Section 1.4. Important Logical Equivalences

A1. [Your choice.] e.g. $x > 50 \Rightarrow x > 10$.

A3. T

^^^^A6-11

A6 and more. [Use truth tables, or one wide truth table.]

| $H$ | $C$ | $H \Rightarrow C$ | not$(H \Rightarrow C)$ | not $C$ | $H \wedge (\text{not } C)$ | $H \wedge (H \Rightarrow C)$ | $H \wedge (H \Rightarrow C) \Rightarrow C$ |
|---|---|---|---|---|---|---|---|
| T | T | T | F | F | | | |
| T | F | F | T | | | | |
| F | T | T | F | | | | |
| F | F | T | | | | | |

A7. Requires 5 columns.

A9. Requires 5 columns.

A11. Requires 4 columns.

A13. If $|x| \le 7$, then $x \le 7$.

A15. If $x \notin$ T, then $x \le 5$.

A17. If it is not over 75 years old, it is not an antique chair.
[or] If the chair is not over 75 years old, it is not an antique.

A19. If $x = 3$, then $f(x) = 0$, and if $x = 8$, then $f(x) = 0$.

A21. If you are at least 65 years old, take this deduction, and, if you are blind, take this deduction.

A23. If you complete the course requirements, you graduate, and if you graduate, you complete the course requirements.

A25. Yes.

A27. True. The converse of $A \Rightarrow B$ is $B \Rightarrow A$, and its converse is $A \Rightarrow B$.

A29. No. ($x = 2$ and $c = -2$)

^^^^^^^^

B2. [7 columns. This is a very important theorem.]

B3. [Use a truth table with 8 rows and 8 columns.]

B4. [Use a truth table with 7 columns and four rows.]

B5. a) and b) are converses; c) and d) are converses; a) and c) are contrapositives; b) and d) are contrapositives

B7. a) See Figure 1.3.3 and note that the set corresponding to $A$ is a subset of the set corresponding to $A$ or $B$. b) $S \subset S \cup T$

B9. See Figure 1.3.8. b) The region for "(not $H$) or $C$" is everything. c) T1.3.13.

B11. If $c > b > 0$, then $|c| > |b|$.

B13. If $x > 9$ and $f(x) = 3x + 5$, then $f(x) > 32$.

B15. If two angles are vertical, then they are congruent.
If two angles are not congruent, they are not vertical.

B17. If the student is a math major, the student is smart.
If the student is not smart, the student is not a math major.

B19. If the car is new, it has an airbag.
If the car does not have an airbag, it is not new.

B21. If the complement of a set is not open, the set is not closed.

B23. If $f(x) \ge f(z)$, then $x \ge z$.

B25. Use a truth table with 9 columns.

B27. See Figure 1.3.8 [*subset* or *if..., then...*]. The region of $H$ being inside the region of $C$ is not the same as the region for $C$ being inside the region for $H$.

## Section 1.5. Negations

A1. a) $c \le 9$ and $c \ge 1$, which can also be written $1 \le c \le 9$.
b) $x \le -5$ or $x \ge 8$.
A3. a) $x \le 25$ and $x \ge 5$, which can also be written $5 \le x \le 25$.
b) $x \le 8$ or $x \ge 32$.

A5. He is not dead and not in jail.
A7. She is not tall or not smart.
A9. If it is small and I cannot lift it, it is not light.
[or] If it is light and I cannot lift it, it is not small.
A11. If $0 < x$ and $x^2 \ge z^2$, then $x \ge z$. [or] If $x < z$ and $x^2 \ge z^2$, then $x \le 0$.
A13. If $c < 0$ and $ca \le cb$, then $a \ge b$.

A15. Fords that are not great trucks do not have four-wheel drive.
A17. If they do not win, they will go home.
A19. If you do not do it today, you must do it tomorrow.
A21. $x \le 5$ or $x^2 < 25$.

A23. If they do not score this possession, then they will lose the game.
A25. When I go on vacation and do not go to the ocean, I go to the mountains.
[or] When I go on vacation and do not go the mountains, I go to the ocean.
A27. If $xy = 0$ and $x \ne 0$, then $y = 0$.
A29. If $xy > 0$ and not($x > 0$ and $y > 0$), then $x < 0$ and $y < 0$.
If $xy > 0$ and ($x \le 0$ or $y \le 0$), then $x < 0$ and $y < 0$.
A31. Treated bacteria that are not dead are dying.
[or] Treated bacteria that are not dying are dead.
A33. No. ($A$ and $B$) $\Rightarrow C$. ($A \Rightarrow C$) and ($B \Rightarrow C$).
A35. Yes. ($A$ or $B$) $\Rightarrow C$. ($A \Rightarrow C$) and ($B \Rightarrow C$).
A37. In the "not both" version one or the other could be 0, but not in the "both not" version.
[e.g. $a = 1$ and $b = 0$ satisfies "not both 0" but does not satisfy "both not 0."]
A39. [similar to A40 with "not($A$ and $B$)".
A41. [9 columns]

| $A$ | $B$ | $C$ | $A \wedge B$ | $A \wedge B \Rightarrow C$ | not $C$ | $A \wedge$(not $C$) | not $B$ | $A \wedge$(not $C$)$\Rightarrow$(not $B$) |
|---|---|---|---|---|---|---|---|---|
| T | T | T | T | T | F | F | | |
| T | T | F | T | F | T | T | | |
| T | F | T | F | T | F | | | |
| T | F | F | F | T | T | T | T | T |
| F | T | T | F | T | | | | |
| F | T | F | F | T | T | F | | |
| F | F | T | F | T | | | | |
| F | F | F | F | T | | | | |

Columns 5 and 9 prove it.

A43. [8 rows and 8 columns]
A45. [4 rows 6 columns]
A47. [8 rows and 8 columns]
A49. [2 rows and 3 columns]
A51. True, it is a tautology.

^^^^^^^^

B1. The negation of "$H \Rightarrow C$" is "$H$ and not $C$." The original says, if $H$ is true, then so is $C$.
The negation says that is false, so $H$ must be true but $C$ not.
The original is false iff $H$ is true and $C$ is not.
The original says nothing about $C$ if $H$ is false, so the original cannot be false if $H$ is false.

B5. a) If $x + y > 2$ and $x \le 1$, then $y > 1$.
b) If $x \le 1$ and $y \le 1$, then $x+y \le 2$,

B7. If $|xy| > 100$ and $|x| \le 10$, then $|y| > 10$.
If $|x| \le 10$ and $|y| \le 10$, then $|xy| \le 100$.

B9. "not($B$ or $C$) $\Rightarrow$ not(not $A$)" is LE to "(not $B$) and (not $C$) $\Rightarrow A$" by DeMorgan's law and double negation.

B11. Substitute "$H \Rightarrow C$" for "$A$" and simplify with 1.5.2.

B13. a and c. b, d and e.

B15. Assume $A$ is true. $B$ is true or false. If $B$ is true, "$A \Rightarrow (B$ or $C)$" is true since its conclusion is. If $B$ is false, not $B$ is true, and then "($A$ and not $B$) imply $C$, so $C$ is true and the conclusion of "$A \Rightarrow (B$ or $C)$" is true, so the conditional is true. It is true in either case, which implies it is true.

B17. The book is not rare and the book is old.
The book is not both rare and old.
The second, The book is not (rare and old).

[B19-21 Venn diagrams omitted]

B19. Sketch 5 figures. $A$ and $B$ (Fig. 1.3.2), not($A$ and $B$), not $A$, not $B$, (not $A$) or (not $B$).

B21. [Venn diagram for $B$ or $C$ being always true.] $A$ or $B$ LE (not $A$) $\Rightarrow B$. [Theorem on Or]

B23. "$H \Rightarrow C$ " iff "(not $C$) $\Rightarrow$ (not $H$)" [is a tautology]

B25. "not($A$ and $B$) iff (not $A$) or (not $B$)" [is a tautology]

B27. True. If $A$ is true and $A \Rightarrow B$ is true, then $B$ is true and $A \Rightarrow$ (not $B$) is false.

B29. not complement, and $\cap$, or $\cup$, if...then $\Rightarrow$, iff $=$

B31. $R \subset (S \cap T)$ iff $R \subset S$ and $R \subset T$.

B33. Let $H$ be a subgroup of $G$. If the left cosets $aH$ and $bH$ are not equal, they are disjoint.
If the left cosets $aH$ and $bH$ are not disjoint, they are equal.

B35. Make $A$ not inside $B$ but $A$ inside ($B$ or $C$). One easy was it to put $A$ inside $C$ and $B$ elsewhere.

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## Section 1.6. Tautologies and Proofs

A1. A proof of the contrapositive proves the statement because the contrapositive is logically equivalent to the statement and has the same truth value as the statement. Therefore, a proof that one is true proves the other is true.

A3. T

A5. True, by Cases.

A7. True. A contradiction is F in all rows. Negating it produces T in all rows.

A9. a) neither b) neither c) contradiction d) tautology

A11. It is LE to "$A \Rightarrow B$ and $A \Rightarrow C$."

A13. $A$ and $B$ A15. $B \Rightarrow$ (not $A$) A17. $A \wedge B \wedge$(not $C$)

A19. $A$ iff $B$ iff $C$ A21. $H \Rightarrow D \Rightarrow E \Rightarrow F$;

A23-30 Truth tables omitted. xx

A31. $H \Rightarrow C$ iff (not $H$) or $C$.
A33. not($H \Rightarrow C$) iff $H$ and (not $C$).

A35. T A37. F
A38. T A40. T
A41. T A43. F
A44. T A46. T
A47. T A49. T

A51. [Your choices] a) $3x > 15$, $x > 5$. b) $x^2 + 3 > 0$.

^^^^^^^^
B1. Only d. (The contrapositive)

B3. False. Think of "Cases." Consider F,T,F. (We don't know that $B$ would imply $C$.)
[($A$ or $B$) $\Rightarrow C$ is LE to $(A \Rightarrow C)\wedge(B \Rightarrow C)$. The "$B \Rightarrow C$" part is missing.]
B5. False. Think of "Cases." [The "$B \Rightarrow C$" part is missing.]
B7. a) nothing b) John is quick.
c) nothing firm, by itself. If he is not quick, he is not a pro basketball player.
[or] He is quick or not a pro basketball player.
d) nothing firm. e) John is not a pro basketball player. f) nothing
B9. a) nothing firm, but a better answer is: If it has four-wheel drive, it is a great truck. [or] If it is not a great truck, it does not have four-wheel drive. [or] It does not have four-wheel drive or it is great.
b) The truck does not have four-wheel drive. c) nothing
d) nothing e) The truck is either not a Ford or not four-wheel drive.
B11. a) nothing firm, but a better answer is: If it is in prime time, then it makes lots of money. [or] If it does not make lots of money, then it is not in prime time. [or] It is not in prime time or makes lots of money.
b) nothing c) It is not a sitcom in prime time. d) It is not a sitcom.
B13. a) $y < 4$. b) nothing c) nothing d) $x \leq 3$.
B15. a) nothing b) $x \leq 5$ c) nothing d) $y > 9$

B17. Yes. B19. No. B21. No.

B23. a) $H \Rightarrow C$ b) (not $C$) $\Rightarrow H$ c) (not $C$) $\Rightarrow$ (not $H$)
d) (not $H$) $\Rightarrow$ (not $C$) e) (not $C$) or $H$ f) (not $H$) or $C$
a, c, and f are LE. Also, d and e
B25. a) $H \Rightarrow C$ b) $C \Rightarrow H$ c) (not $H$) $\Rightarrow$ (not $C$)
d) (not $C$) $\Rightarrow$ (not $H$) e) (not $H$) or $C$ f) (not $C$) or $H$
a, d, and e are LE. Also, b, c and f are LE.

B27. d and e. a) no b) no c) no d) FL e) FL
B29. a) and g) follow logically
B31. a) and b) and f) follow logically.
B33. d and h follow.

B35. [(not $B$)$\wedge$(not $C$) $\Rightarrow$ (not $A$)]. Solution by DeMorgan's:
[not($B$ or $C$) $\Rightarrow$ (not $A$)] iff [(not $B$)$\wedge$(not $C$) $\Rightarrow$ (not $A$)]
B37. A$\wedge$[not ($B\wedge C$)] iff $A\wedge$[(not $B$) or (not $C$)] iff
[$A\wedge$(not $B$)] or [$A$ and (not $C$)] ← the answer
b) Yes. $A\wedge$(not $B$) would do.

B39. a) not[($A$ or $B$) $\Rightarrow C$] iff not[($A \Rightarrow C$)$\wedge$($B \Rightarrow C$)]
iff [not($A \Rightarrow C$)] or [not( $B \Rightarrow C$)] iff
[$A\wedge$(not $C$)] or [$B\wedge$(not $C$] ← the answer
b) Yes. $A\wedge$(not $C$) would do.

B41. $A$ and $B \Rightarrow C$. Proved in the same form.

B43. $A \Rightarrow (B \Rightarrow C)$ [or] ($A$ and $B$) $\Rightarrow C$. Proved as $A$ and not $C \Rightarrow$ not $B$. [A Version of the Contrapositive.]

B45. a) "If $H$, then, if $x \in S$, then $|x| \le 120$."
b) $H \Rightarrow (B \Rightarrow C)$ c) A Hypothesis in the Conclusion
d) $B$ "Let $x \in S$ " e) $|x| \le 120$. f) $H$

B47. $A$ and $B \Rightarrow C$ [or $A \Rightarrow (B \Rightarrow C)$] Proof: $B$ and $A \Rightarrow C$.

B49. $A \Rightarrow B\wedge C$ (hypothesis)
$\Rightarrow B$ (Tautology 1.6.4C)
Therefore $A \Rightarrow B$ (transitivity of " $\Rightarrow$ ")

B51. a) only

B53. b), c) and d) do.

B55ff. omitted