Solutions Manual

Selected odd-numbered problems in

Chapter 2

for

PROOF:
INTRODUCTION TO HIGHER MATHEMATICS

Seventh Edition

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5 4 3 2 1

Chapter 2: Sentences with Variables

Section 2.1. Sentences with One Variable

A1. Open sentences, generalizations, and existence statements.

A3. The true generalizations are: b, c (they use placeholders)
A5. The true generalizations are: a, c (they use placeholders)
A7. The true generalizations are: b (it uses a placeholder)
A9. The true generalizations are: b (it uses a placeholder)
A11. The true generalizations are: a, c (they use placeholders)
A13. Those fitting the convention are: b,c
A15. Those fitting the convention are: a,c
A17. b, d

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B3. An equation is an open sentence -- the truth may depend upon the value of the variable. An identity is a special type of equation, one which is true for all values of the variable. If an equation is true "for all x," it is an identity (and interpreted as a generalization).
Usually, equations are about numbers and identities are about operations and order.

B5. a) No. b) Yes.
B7. a) S b) S, T c) a, b, c
B9. a) generalization b) open c) existence statement
d) open e) generalization
Generalizations and existence statements use placeholders. Open sentences use free variables.

B11-21 abbreviations: OO = operations and order, N = number
B11. a) OO (add 5 and multiply by 3) b) N (x) c) squaring
B13. a) OO (Add 1) b) N (x) c) OO (squaring after taking negatives)
B15. a) (sets) The sets S and T b) OO intersection and subset c) OO complement and intersection
B17. a) *not* and *or* b) *if...,then...* and *and* c) *if...,then...* and *and*
B19. a) set complement and intersection b) the sets S and T c) subset
B21. a) x and c b) the inequality with absolute values "$|x| < c$"
c) real numbers named "x" [with "or" and "not"]
B23. a) no. b) connectives and order. c) placeholder

B25. A tautological sentence is always true, and it is so because of the arrangement of its connectives (and not the meaning of the connected sentences). A statement can be always true for other reasons. For example, it could be true by definition, or because of a theorem.
Example: "$x - 2 < x$" is always true, but there are no connectives and logic is not why it is true.
Example: "$|x| < c$ iff $-c < x < c$" is always true, but not because the connected sentences are *logically* equivalent.

Section 2.2. Existence Statements and Negation

A3. a) 1 and 4 b) 3 and 4 c) 3 and 4
[or] 1) a 2) none 3) b, c 4) a, b, c

Note to problems A5 - A12: Subscripts may be included or omitted. Also, as always, quantified variables may be represented by other letters.

A5. There exists x such that $g(x) > 12$.

A7. There exists x such that $|x| > 7$ and $x \le 7$.

A9. There exists $x \in S$ such that $x > 25$.

A11. There exists $x > 42$ such that $x \in S$.

A13. There exists x such that $f(x) \ne g(x)$.

A15. There exists $x > 6$ such that $f(x) \le g(x)$.

^^^^ The next problems request counterexamples. **The answer here is one among many possible counterexamples**.

A17. Let $x = -1$ [or, any other negative number]

A19. $x = -10$ [or any other number < -5]

A21. $x = -1$

A23. $x = -20$

A25. $a = 0$ and $b = -2$.

A27. Let $x = 1$. Then $x^2 - 2x + 1 = 0$, not > 0.

A29. "Not all batteries are alike."

A31. b, c, e A33. a, e, f

A35. d, e

A37.	a) Y	b) N	c) N	d) N
A39.	a) N	b) N	c) N	d) N
A41.	a) N	b) Y	c) Y	d) Y
A43.	a) Y	b) N	c) N	d) N

^^^^^^^^

B3. a) There exist a and b such that $ab > 0$ and not($a > 0$ and $b > 0$).
[or] There exist a and b such that $ab > 0$ and $a \le 0$ or $b \le 0$.
b) Let $a = b = -1$. Then $ab = 1 > 0$.

^^^^ The counterexamples given below are not the only ones. The student's answer need not (and probably will not) duplicate the one example given.

B5. F. $b = -10$ and $c = -10$.

B7. T.

B9. F. $x = 5$ and $y = -5$.

B11. F. $a = 5$, $b = 10$, $c = 1$, and $d = 10$. 4 is not less than 0.

B13. False. $a = 1$, $b = 2$, and $c = 0$.

B15. Y B17. Y B19. Y B21. N

^^^^ **Negations**

B23. There exists x in S such that $x \le 5$.

B25. There exists $x \in S$ such that x is not in T.

B27. There is a polynomial with exactly three local extrema.

B29. There is a basketball player who scored fewer than 4 points (3 or fewer points).

B31. There is an x such that $f(x) < g(x)$.

B33. There exists a horizontal line that intersects the graph at least twice [more than once].

B35. There exists a vertical line that intersects the graph twice (or more).

B37. All piles have at most one ball. ["No piles have more than one ball" is not positive form.]
B39. There exists a pile with more than 20 chips.

B41. There is a horizontal line that intersects the graph twice or more.
B43. There is an $x \in S$ such that $x > 7$ and x is not in T.
B45. There is an x in S such that $x > 4$ and x is not in R.
B47. There exists a polynomial of degree 3 that does not have 2 local extrema.

B49. All piles have less than 20 chips. [or] All piles have at most 19 chips.
B51. There is a team with a lineman under 230 pounds.
B53. There exists $\varepsilon > 0$ such that for all $\delta > 0$ there is an x such that $0 < |x - a| < \delta$ and $|f(x) - L| \geq \varepsilon$.
B55. There exists a horizontal line which intersects the graph of f twice or more.

B57. There exists x such that x is in S and $f(x) > 6$ and $x \leq 2$.
[or] There exists $x < 2$ such that x is in S and $f(x) > 6$.
B59. There exist a, b, c, and d such that $a \neq 0$ and $d > 0$ and $ax^2 + bx + c = d$ has a solution. [or] There exist $a \neq 0$, $d > 0$, b and c such that $ax^2 + bx + c = d$ has a solution.
B61. There exists $c < y$ such that for all x in S, $x \leq c$.
B63. There is a $k > 0$ such that for all x in S [either] $x = p$ or $|x - p| \geq k$.

B65. There exists n such that $n^2 + n + 41$ is composite [not prime].
B67. a) There exist x and z such that $x < z$ and $f(x) \geq f(z)$. b) Choose $x = -2$ and $z = 1$.
B69. a) For all x there exists y such that $x < y$.
b) Yes, the negation is true. (So, the original is false). Let $y = x+1$.
B71. "Give the negation in positive form of: "For each $x \geq 0$, there exists $y \geq 0$ such that $y < x$." The negation is: There exists $x \geq 0$ such that for all $y \geq 0$, $y \geq x$.
Proof: Choose $x = 0$. Then for all $y \geq 0$, $y \geq x$.

^^^^ **Negations from Calculus**
B73. $\exists\ \varepsilon > 0$ such that $\forall\ \delta > 0, \exists x, t$ such that $|x - t| < \delta$ and $|f(x) - f(t)| \geq \varepsilon$
B75. $\exists\ m$ such that $\forall n^*\ \exists n > n^*$ such that $a_n \leq m$.

B77. True. a) If $x > 1$, then there exists $\varepsilon > 0$ such that $x \geq 1 + \varepsilon$. b) Let $x > 1$. Choose $\varepsilon = x$-1. Then $x = 1+\varepsilon$.
B79. If, for all x in S, $x \leq \sup(S) - c$, then $c \leq 0$.

^^^^ **Nested Quantifiers**
B81. a) yes b) no
B83. a) F b) T
B85. a) F b) T
B87. True. Choose $y = (x+1)/2$.
B89. False. Choose $x = 0$.
B91. True. Choose $y = x$.
B93. True. Choose $y = x/2$.
B95. True. Choose $y = x+1$ [or $2x$]
B97. False. Choose $x = b$.
B99. True. $y = x + 1$.
B101. omitted
B103. Let $y > 4$. then $3y > 12$ and $3y > (3y + 12)/2 > 6$. Choose $x = (3y + 12)$.
B105. True. If $x \neq 0$, then $|x| > 0$. Let $c = |x|$. Then "$|x| < c$ is false. [Proof by contrapositive.]

^^^^ **Negations**
B107. a) $x = 3$. b) There exists x such that $x > 3 - \varepsilon$ for all $\varepsilon > 0$, but $x \leq 1$
B109. There exist a, b and $c < 0$ such that $ax^2 + bx + c = 0$ does not have a solution.
(or, is not zero for all x).
B111. H and (not(B and C)) is LE to H and (not B or not C) is LE to (H and not B) or (H and not C).

B113. For a given x, it is true. But, if "for all x" is attached to both components, it could be false [e.g. $f(x) = x$, for all x.] The key is the location of the quantifier(s).

B115. a) If M and A and B, then Z. b) If M, then (not A) or (not B) or Z
so, If f has a local max at c, then $f'(c) = 0$ or $f'(c)$ does not exists or c is not in (a, b).

Section 2.3. Reading Theorems and Definitions

A3. a) $1 + 2 + 3 + ... + n$ b) $n(n + 1)/2$.

A5. 200(201)/2 = 20100

A7. 1000(1001)/2 = 500,500

A9. $$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-5)}}{2(1)} = \frac{-3 \pm \sqrt{29}}{2} = 1.19 \text{ or } -4.19$$

A11. $$x = \frac{-1 \pm \sqrt{1^2 - 4(-1)(4)}}{2(-1)} = \frac{-1 \pm \sqrt{17}}{-2} = -1.56 \text{ or } 2.56$$

A13. a) $x \in S^c$ b) No, it is open. c) The definition of it is open
The definition in sentence-form is true (by definition).

A15. a) b is a bound of S. b) noun

^^^^ **Upper bounds and bounds**

A17. a) true b) false

A19. If $x \in (1,3)$, then $x \le 35$.

A21. If $x \in [-45,23)$, then $|x| \le 50$. .

A23. 3 is one such upper bound. Any b, $1 \le b < 5$ would do.

A25. a) $[3, \infty)$ b) $[3, \infty)$ c) $[4, \infty)$ d) Yes, they are *in* the set of upper bounds.

A27. 10 is an upper bound of (-20, 9) but not a bound of it.

A29. b, f

^^^^ **Interior points**

A31. a) true b) false

A33. a) yes b) no c) yes d) yes e) no

A35. If $x < z$, then $x^3 < z^3$

A37. $[3(x+h)+5 - (3x+5]/h = 3h/h = 3$ if $h \neq 0$.

A39. $[-5 \pm \sqrt{(25 - 4(1)(-2))}]/2$

A41. $[-7 \pm \sqrt{(49 - 4(-2)(12))}]/(-4)$

A43. If $x < z$, then $\sqrt{x} < \sqrt{z}$ (if x and z are in the domain, that is, $x \ge 0$).

A45. $|x| \ge 5$ or $x^2 < 25$.

^^^^ **Logic**

A47. a) no b) connectives. [the relationship of *not* and $\Rightarrow$ (in a certain order).]
c) placeholder

A49. (not C) $\Rightarrow$ not(A and B);

A51. (not A) $\Rightarrow$ not(not B) which is LE to (not A) $\Rightarrow$ B.

A53. $A \wedge \text{not}(B \wedge C)$
A55. $A \wedge \text{not}(B \text{ iff } C)$.

A57. [several answers are possible] $(B \Rightarrow C)$ and $(B \Rightarrow A)$
A59. $(B \Rightarrow D)$ and $(C \Rightarrow D)$
A61. not C and not B

A63. $(H \Rightarrow B)$ and $(B \Rightarrow H)$
A65. $H \wedge(\text{not C})$
A67. C and not $A \Rightarrow B$
A69. $(C \Rightarrow A) \wedge (C \Rightarrow B)$

^^^^ **Theorem 1**

B3. a) $j(j+1)/2$. b) $(k+1)(k+2)/2$.
B5. a) $1000(1001)/2 - 99(100)/2$.
b) $n(n+1)/2 - (k-1)k/2$ [You may stop here.]
$= (n^2 + n - k^2 + k)/2 = (n^2 - k^2 + n + k)/2 = (n - k + 1)(n + k)/2$.
B7. a) $2n - 1$
b) $1000^2 = 1{,}000{,}000$. [reasoning next]
$1+2+3+ \ldots + n = n(n+1)/2$, by Theorem 1
$2+4+6+ \ldots + 2n = n(n+1)$ [multiplying by 2]
$1+3+5+\ldots + (2n-1) = n(n+1) - n = n^2$. [subtracting 1 for each term on the left]
c) the sum of $2k - 1$ is the 2 times sum of k minus the sum of 1
$= 2n(n+1)/2 - n = n^2$
[or] $1 + 3 + 5 + \ldots + (2n - 1)$ is n^2, by inspection, which is not a proof or derivation.
[or] The sum of all integers up to $2n$ minus the even ones. $2n(2n+1)/2 - 2n(n+1)/2 = n^2$.
[or] It is all integers up to $2n - 1$ minus the even ones
$1 + 2 + 3 + 4 + \ldots + (2n - 1)$ - (a sum of n-1 even integers)
$= (2n - 1)(2n)/2 - (n - 1)(n)$ [from Theorem 1, times 2] [or]
$= n(2n - 1 - (n - 1)) = n^2$. [This can be also proved by mathematical induction.]
d) $[(k+1)/2]^2$, because $k = 2n - 1 \Rightarrow n = (k+1)/2$.

^^^^ **The Quadratic Theorem**

B9. $x = [-2b \pm \sqrt{(4b^2-4c)}]/2$
B11. $x = [-a \pm \sqrt{(a^2-4cb}]/2b$
B13. $x = [-b \pm \sqrt{(b^2 + 4ac)}]/2a$
B15. $y = [8x \pm \sqrt{((-8x)^2 - 4(1)(x^2 + x - 20))}]/2$

B17. Treat x^4 as $(x^2)^2$. $x^2 = [12 \pm \sqrt{(144-48)}]/6$; x is $\pm\sqrt{}$those two. $= \pm 1.906$ or ± 0.606
B19. Treat x as $(\sqrt{x})^2$. $\sqrt{x} = [-1 \pm \sqrt{(1+400)}]/4$. The negative is impossible as a square root,
so $x = \{[-1+\sqrt{(1+400)}]/4\}^2 = 22.6$.
B21. $5^* = 2$. $8^* = 4$. Thus the problem is to solve $n^* = 2(4) = 8$.
The solution is $n = 16$ or $n = 17$.
B23. $n^* = (3^*)(6^*) = 9(3) = 27$, so $n = 9$ or $n = 54$.
B25. $(-4)' = 12$. $2' = 4$. So the equation is $n' = 48$. $n = 24$ or -16.
B27. $f(5) = 15$. $f(14) = 7$. $f(n) = 15(7) = 105$. $n = 35$ or 210.

B29. a) $3@7 = 2(7) - 3^2 = 5$. b) $5@a = 2a - 25$
c) $x@4 = 7@x$. $2(4) - x^2 = 2x - 7^2$. $x^2 + 2x - 57 = 0$. Solve with QF:
$x = 6.62$ or $x = -8.62$.
B31. a) $x^2 - 2 = (16-2)(9 - 2) = 98$. $x^2 = 100$. $x = \pm 10$.

B33. True. Let $x < z$. Then $g(x) \le g(z)$ because g is increasing. Then $f(g(x)) \le f(g(z))$ because f is increasing.

B35. a) f is decreasing iff $x < z \Rightarrow f(x) \ge f(z)$. b) False. x^2 is neither.

^^^^ **Upper bounds and bounds**

B37. True.

B39. a) F, the reals b) F, $(-\infty, 2]$ c) T d) T

B41. a) Yes. b) No.

B43. a) m is a lower bound of T iff $x \in T \Rightarrow m \le x$.
b) T is bounded below iff there exists a lower bound of T.
c) your choice d) True. (3, 7] has no least element in it.

B45. True. (By contradiction) Suppose S is bounded and S^c is bounded. Then, by T16, $S \cup S^c = \mathbb{R}$ is bounded, but it is not.
[or] If S is bounded, then S^c is not.
Proof: There exists b such that $x \in S$ satisfies $|x| \le b$, by hyp. So if $|x| \ge b$, then x is not in S. Therefore, for any d, there exists $x \in S^c$ such that $|x| > d$, so S^c is not bounded.

B47. False. If S is bounded, then S^c is not.
(All numbers larger than the bound of S will be in S^c.)
Proof. If both were bounded, so would their union be, by Theorem 16, but their union is the set of all reals, which is not bounded. So, by contradiction, they cannot both be bounded.

B49. This is LE to: If n is even, then n^2 is even. Proof: n is even means there is k such that $n = 2k$. Then $n^2 = (2k)^2 = 2(2k^2)$ is even.

^^^^ **Interior points**

B51. Any open interval about 5.2 would have to contain points to the right of 5.2, which would not be in [0, 5.2].

B53. F 2 is not an interior point of $\mathbb{N}$

B55. T (use the same delta)

B57. F 2 and the rationals

B59. if. only if.

B61. n is divisible by 3 iff there exists j such that $n = 3j$.

B63. a) $(3,5)\bullet(2,9) = 3(2)+5(9) = 51$. b) $3(2x)+x(5) = 20$. $11x = 20$. $x = 20/11$.

B65. a) $[1-2^{k+1}]/(1-2) = 2^{k+1} - 1$ b) $(1-x^9)/(1-x)$, if $x \ne 1$.

B67. $-3\pi \sin(\pi x)$

B69. $-13(1/2)\sin(x/2)$

B71. $-(1/7)(1/2)\sin(x/2) = -\sin(x/2)/14$

B73. $14 \cos(2x)$

B75. $(11/3)\cos(x/3)$

B77. $\cos(x/5)/15$

B79. $x \in S\text{-}T$ iff $x \in S$ and $x \notin T$.

B81. b is a lower bound of S iff, $x \in S \Rightarrow b \le x$.

B83. $50^2 = 2500$

B85. Let $k = 2n - 1$. $n = (k + 1)/2$. The sum is $((k + 1)/2)^2$.

B87. a) $\{\varnothing, \{1\}, \{2\}, \{1,2\}\}$ b) $\{\varnothing, \{3\}\}$
c) If $R \in P(S)$ then $R \subset S$, by the definition of power set. Then $R \subset T$, by hypothesis. Then $R \in P(T)$, by the definition of power set, in other direction.
B89. a) $2^3 + 2 = 10$ b) $2x = 3^3 + 3 = 30$. $x = 15$

B91. a) {1, 2} b) {6, 7}
B93. 120
B95. $x = 20$
B97. -2
B99. $8a - 2bc$
B101. $15 - 2x = 3$. $x = 6$
B103. $x^2 - 16 = 9$. $x = \pm 5$

B105. a) (6, 8, 0) b) $(d, a, 0)$ c) $(c, 0, 0)$

B107. $A \wedge B \wedge (\text{not } C) \Rightarrow D$ "Or" in the Conclusion
B109. H and $B \Rightarrow C$ "Or" in the Conclusion
B111. a) A and (B or C) $\Rightarrow D$ b) $\{B \Rightarrow [A \Rightarrow D]\}$ and $\{C \Rightarrow [A \Rightarrow D]\}$

B113. a) not A or not(B or C) b) not[(A and B) or (A and C)]

B115. a Hypothesis in the Conclusion; Two Conclusions
Contrapositive (in two different places); a Version of the Contrapositive
B117. $15x^4$
B119. $(2.3)12x^{1.3} = 27.6x^{1.3}$

B121. $2 \otimes 3 = 6$. $1 \otimes 1 = 2$. $4 \otimes 3 = 4$. So the right side is 48.
$x \otimes 24 = 48$. So $x = 16$ or 24 or 48 (one answer from each of the three cases).

B123. There exists $\varepsilon > 0$ such that for all $\delta > 0$, there exists x such that $0 < |x - x_0| < \delta$ and $|f(x) - L| \geq \varepsilon$.

B125. ... iff there exists $\varepsilon > 0$ such that for all x in S, $x = p$ or $|x - p| \geq \varepsilon$.
B127. a) There exists $\varepsilon > 0$ such that $(p - \varepsilon, p + \varepsilon)$ is either a subset of S or a subset of S^c.
b) The existence of $\varepsilon = .1$ (or smaller).
B129. a) T b) F c) T d) T
B131. T
B133. F. Counterexample: 1 is a boundary point of (0,1) and (1, 2).
B135. F. Counterexample: (1, 2) is a subset of (0, 3).

Section 2.4. Equivalence

A1. a) False b) True c) True

A3. a) True b) True c) False

A5. a) false b) false
A7. a) F b) T

A9. a) F (but close. T if $c \neq 0$.) b) T
A11. a) F b) F $x = -7$ [$\sqrt{-7}$ does not exist as a real number]

A13. a) T b) T

A15. a) $x = -1$, b) $x = 1$
c) For each x, $2x > 0$ or $2x \le 0$. [x's considered one at a time]
A17. F. One counterexample: Let $S = \{1, 2\}$ and $T = \{2\}$.
A19. a) There exist a, b, and c such that $ab = ac$ and $b \ne c$. b) $a = 0$, $b = 1$, and $c = 2$.
A21. There exists $x \in$ S such that $x \notin$ T.

A23. There exists $x \in$ S such that $x > 9$.
A25. a) Yes b) No c) No d) Yes e) No f) Yes
A27. If $x \notin T$, then $x \notin S$.
A29. If $x > 9$, then $x \notin S$.
A31. F. $x = 4$, $c = -2$.

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B3. For example, define $f(x) = 0$ if $x \le 2$ and $f(x) = 1$ if $x > 2$.
Define $g(x) = 3$ if $x > 1$ and $g(x) = 0$ if $x \le 1$.
Then, for each x, either $f(x) = 0$ or $g(x) = 0$. But neither is 0 for all x.
B5. a) It is a tautology, for fixed x.
b) It is possible for "For all x, $A(x)$" and "For all x, not($A(x)$)" to both be false (as in B3).
B7. a) A or $B \Rightarrow C$ b) ($x > 5 \Rightarrow |x| > 5$) and ($x < -5 \Rightarrow |x| > 5$)

B9. a and c. b and d. e) b and d are true.
B11. c and f. b, d and e.

B13. a) Yes b) No. c) Yes. d) No.
B15. a) Yes, FL b) No. not FL c) No. not FL d) No. not FL e) No. not FL
f) No. not FL g) Yes, FL
B17. a) FL b) FL c) N d) N e) N
f) N g) FL
B19. All "no" except the last one, h.
B21. converses: b and h. c & g

B23. contrapositives: theorem & c

B25. a) FL b) FL c) no d) no [should be "or"]

B27. a) if it is non-Abelian, then it is of even order. [or] It is Abelian or of even order.
b) nothing
c) If it is simple, it is of even order [or] It is not simple or of even order
d) it is not (non-Abelian and simple) [it is Abelian or not simple]
e) nothing f) nothing

B29. F ($x = 1$) B31. F ($x = y+1$)

B33. a) FL b) FL c) not FL d) FL
B35. a) S and T have the same members. b) $x \in S$ iff $x \in T$
c) $c \in S$ iff $c \in T$ d) $S = T$
e) $x \notin S$ iff $x \notin T$ e) $S^c = T^c$

Section 2.5. Rational Numbers and Form

A3. No. Since Conjecture 3 is false, maybe consider: "If x and y are both irrational, then xy is rational." [We can disprove that using results from Section 3.4 about $\sqrt{2}$.]

A5. a) If x is irrational and xy is rational, then y is rational. b) This is false.

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B1. If the form fits one of the theorems about LE in Chapter 3, it may be proved in the alternative form. Some LE forms may be more natural because they come closer to the form of the relevant definition or prior result.

B3. False. Let $x = y = \sqrt{2}$.

B5. This argument is incorrect. Not (always irrational) is not the same as (always rational)

B7. "If x is rational and xy is rational, then y is rational."
This is LE to Conjecture 4! It's false. See B4. $x = 0$ and $y = \sqrt{2}$.

B9. False. $y = 0$ and $x = \sqrt{2}$. It is LE to "If y is rational and xy is rational, then x is rational."

B11. False. $x = \sqrt{2}$ and $y = 3$.

B13. a) There are counterexamples to both Conjectures 10 and 11.
b) One says, for all x and y, $H(x, y) \Rightarrow C$. The other says, for all x and y, $H(x, y) \Rightarrow$ not C.
The problem is that when the hypothesis is true, C is true sometimes, but not always.
c) [your choice of new example]

B15. True. Because x and y are rational, there exist p, q, m, and n, where q and n are not 0, such that $x = p/q$ and $y = m/n$. Then $x + y = (pn + qm)/(qn)$ where the denominator is not 0, so $x + y$ is rational.

B17. True. This is B16 with different letters.

B19. False. $\sqrt{2} + -\sqrt{2} = 2\sqrt{2}$ is irrational [by "Conjecture 4, fixed."]

B21. True. This is a Version of the Contrapositive of B16.

B23. "If $x+y$ is irrational, then x is irrational or y is irrational" by B22.

B25. True, by Conjecture 2.

B27. True. This follows from the contrapositive and Conjecture 2.

B29. True. Let $y = x/2$.

B31. False. $x = y + 1$ is a counterexample.

B33. True. Let $y = x/2$.

B35. False. Let $x = y$.

B37. It proves it.