Math 242. Fall 2014 Name _____
Exam on Chapter 1 of *Proof*.

1. (4 pts) Write out the pronunciation of "$(5, 7) \subset \{x \mid x^2 > 4\}$." [Do it so someone who was listening but not looking could write this down.]

2. (4 pts) These might have grammatical mistakes, given our conventions. If they do not, say "okay." If they do, note what is wrong with them.
a) $[5] \subset [2, 12]$ b) $x \in S$ and $x \in T \Rightarrow S \cap T$.

c) $\{0, 7\} \cap (3, 19)$ d) $3x < 6 \Rightarrow x \in (-\infty, 10)$

3. (3 pts) Compute and simplify these using interval form, as much as possible:
a) $\{2, 5\}^c$. b) $(3, 9) \cap [4, 20]$ c) $\{x \mid x^2 > 16\}$

4. (6 pts) True or false? (For each, circle T or F. No reason required.)

a) T F If "$A \Rightarrow C$" is true, then "A and $B \Rightarrow C$" is true.

b) T F If "(A or B) $\Rightarrow C$" is true, then "$A \Rightarrow C$" is true.

c) T F $x > 5 \Rightarrow x \geq 5$.

d) T F $n > 5 \Rightarrow n \geq 6$.

e) T F $(1, 4) \subset [2, 9]$

f) T F $S \cup T \subset R \Rightarrow S \subset R$.

5. (2 each, 6 pts) Here is a form: $H \Rightarrow C$.
a&b) Give two other forms that are logically equivalent to it.

c) Give its converse.

6. (5 pts) There are five logical connectives and five corresponding set-theory terms. List the five connectives along with the corresponding set-theory term of each. [List five pairs.]

a) b) c) d) e)

7. (4 pts) Give the definitions (symbolically, in sentence-form, not just in English):
a) *set union*

b) *set equality (=)*

8. (4 pts) a) Define *contradiction*.

b) State one.

9. (6 pts) Are these pairs of sentences equivalent? If so, just say "Yes". If not, say "No" and give a **specific** counterexample. [Assign particular values to each letter.]
a) $x^2 > 5x$ $x > 5$.

b) $x < 7$ $2y < 14$

c) not($x < -4$ or $x > 6$), $x \geq -4$ and $x \leq 6$.

10. (12 pts) Rewrite each statement using the logical equivalence we studied that addresses the form. Also, cite **our name** for the equivalence you use.

a) ($x \geq 0$ and $x^2 > 9$) => $g(x) > 5$.

b) ($x > 8$ or $x < 2$) => $f(x) \geq 6$.

c) $x > 2$ or $f(x) \leq 10$.

d) If $f(x) > 6$, then $x < 1$ or $x \geq 7$.

e) If $f(x) > 6$, then $x < 1$ or $x \geq 7$. [This is (d) again. Do it again, another way.]

f) If $S \cap T = \varnothing$, then ($x \in S$ => $x \notin T$).

11. (4 pts) Mathematicians define "$H \Rightarrow C$" to be true whenever H is false. Why? Explain this as if to a student who does not know. [A few correct statements with a few of the right words will not get full credit unless it is a good **explanation**. A truth table does not explain itself, so don't cite the truth table!]

12. (4 pts) We showed "$S \subset T$ or $S \subset T^c$ " is false, with a counterexample. For this problem, give, instead,

a) a clearly-labeled Venn diagram
which illustrates why it is false.

b) a clearly labeled number-line picture which illustrates why it is false.

13. (6 pts) Create a truth table, with all appropriate columns (Do not skip columns!), for determining if "(not A) => B " is logically equivalent to "A or not B ." At the end, say if they are or are not logically equivalent, and why.

14. (6 pts) If you know the following conditional is true, state what you can deduce with the given additional fact.

If a set is twing, then it is secall.

a) S is secall. Deduce:

b) S is not secall. Deduce:

c) S not twing. Deduce:

15. (10 pts) **Instructions**: Simply determine which conjectures **follow logically** (FL) from Assertion 1, or not (N). Assertion 1: If $x > 7$, then $f(x) \geq 4$.

a) FL N $f(5) \geq 7$

b) FL N If $f(x) < 2$, then $x < 9$.

c) FL N If $f(x) < 8$, then $x \leq 7$.

d) FL N If $|x| > 8$, then $f(x) > 1$.

e) FL N If $|f(x)| < 3$, then $x < 8$.

16. (8 pts) Suppose this is a fact: "If $x < 2$ or $x > 6$, then $f(x) < 9$."
If the following is also a fact, what can be deduced from those two facts?

a) $x < 5$ and $x > 4$.

b) $x < 6$ and $f(x) = 11$.

c) $f(x) > 13$.

d) $f(x) = 9$ and $x > 4$.

17. (8 pts) Suppose this is true: "If a yrk is frid and not coley, then it is bruk."
What follows logically from these additional facts?

a) The yrk is coley and not bruk.

b) The yrk is frid and not bruk.

c) The yrk is not bruk and not coley.

d) The yrk is frid or not coley.