Proof. Fall 2014. Exam on Chapters 1 and 2 of *Proof*. Name ________________________

1. (6 pts) Complete these logical equivalences the way they were completed as theorems in the text:

a) A => (B => C) is logically equivalent to

b) (A or B) => C is logically equivalent to

c) A => (B or C) is logically equivalent to

2. (6 pts) There are only three types of sentences with one variable, x. Name them and give a good example of each, being very clear which example is of which type.

3. (5 pts) Solve for y: $2xy + 5y + 5y^2 = 7x + 2$. [Just solve it; do not simplify.]

4. (8 pts) True mathematical sentences convey information about something. Assuming these are true, what is their information about?

a) $3(x + 5) = 3x + 15$

b) $3(x + 5) = 33$

c) $S \subset S \cup T$

d) A or not(A)

5. (21 pts) Give the negation in positive form of

a) If $b > 0$ and $a^2 > b^2$, then $a > b$.

b) [Let f and a be given] For $\varepsilon > 0$ there exists $\delta > 0$ such that if $|x - a| < \delta$, then $|f(x) - f(a)| < \varepsilon$.

c) [Let S and p be given] For each $\varepsilon > 0$ the open interval $(p - \varepsilon, p + \varepsilon)$ has at least one point in S and at least one point not in S.

d) [About several piles of chips] At least one pile has at most 6 chips.

e) Rejegs of degree three have at least five hoppoints.

f) $c = mx + b$ has a solution when $b \neq 0$.

g) If, for all $c > 0$, $x < k + c$, then $x < k$.

6. (6 pts) Define n^ this way: n^ = $6n$ if n is not divisible by 3 and n^ = $n/3$ if n is divisible by 3. Solve for n: n^ = (4^)(9^) [where the parentheses indicate regular multiplication].

7. (6 pts) Give three sentences equivalent to "S is a subset of T", each for a different reason.

a)

b)

c)

8. (8 pts) True or false [No reason required here, but see the next problem]

a) T F For all x in [4, 6) there exists y in [4, 6) such that $y < x$.

b) T F For all x in [4, 6) there exists y in [4, 6) such that $y > x$.

c) T F There exists y in [4, 6) such that for all x in [4, 6), $y \le x$.

d) T F There exists y in [4, 6) such that for all x in [4, 6), $y \ge x$.

9. (4 pts) At least one of the parts of the previous problem is true. Pick a true part (be very clear which part you are picking; they look a lot alike) and prove it true.

My proof is of part ____

Proof:

10. (8 pts) Suppose this is true: If $x > 5$, then $f(x) \le 9$. Which of these follow logically (FL)?

a) FL not FL $f(6) < 12$.

b) FL not FL If $|x| > 6$, then $f(x) < 10$.

c) FL not FL If $|f(x)| < 4$, then $x > 3$.

d) FL not FL If $x > 6$ and $x < 9$, then $f(x) < 10$.

11. (8 pts) Suppose this is true: "If $x > 4$ and $y < 5$, then $f(x, y) < 8$."
What follows logically, if anything, from that and this additional fact?

a) $f(x, y) = 10$ and $y > 7$.

b) $x > 5$ and $f(x, y) < 7$.

c) $y < 5$ [Give a deduction with "or" in it]

d) $x = 5$ or $y = 3$

12. (4 pts) "If S is bounded, either $\sup(S) \in S$ or $\sup(S)$ is an accumulation point of S."
Rewrite this as an equivalent sentence using the logical equivalence we have that fits its form.

13. (10 pts) Prove or disprove this conjecture from the definition (do not cite results we proved)
Conjecture: If a is irrational, then b is irrational or $a+b$ is irrational.