Exam on Chapter 3 of *Proof*. Name:___________________________
Fall 2014 initial each sheet of extra paper

1. (12 pts) True or false? [No reason required here, but see #2.]
a) T F If $a < b$ and $a \neq 0$ and $b > 0$, then $1/a < 1/b$.
b) T F If $|x| < |y|$ and $c > 0$, then $|x - c| < |y - c|$.
c) T F If $|x - c| < d$, then $|x + c| < |c| + d$.
d) T F If $|x^2 - c^2| < 1$, then $|x - c| < 1$.

2. (4 pts) At least one of the above is false. Select one (be clear which one) and prove it false.

3. (16 pts) Give the **negation, in positive form**, of each of these [See also problems 4 and 5.]

a) S is bounded.

b) If $a < 0$ and $c < 0$, then $ax^2 + bx + c = 0$ has no real-valued solution.

c) [f is given.] For each $m > 0$ and for all c, there exists $x > c$ such that $f(x) > m$.

d) [S is given.] For each x in S there exists y in S such that $y < x$.

4. (6 pts) This refers to part (b) of problem 3.
Prove this is false: "If $a < 0$ and $c < 0$, then $ax^2 + bx + c = 0$ has no real-valued solution."

5. (8 pts) Refer to part (d) of problem 3: "For each x in S there exists y in S such that $y < x$."
a) Give an example where that original sentence is true.

b) Give an example where that original sentence is false.

[over]

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Demonstrate that you know how proofs and disproofs are written. If you did it on the homework, or it was done in class or in the text, **do it again here**.

Resolve conjectures with proofs or disproofs. Do not cite results to prove themselves, make sure your steps are prior. **Cite justifications!** (of work at the level of the problem–but not for algebra.) Do work on separate sheets provided.

Correct work without reasons for the steps (unless they are much lower level) **will not get full credit**.

Do work on separate sheets provided.
(12 points each for the next two).

6. Prove by induction: $6(10^n) - 15$ is divisible by 9 for all $n \geq 1$.

7. [For this one, nothing about rational and irrational numbers is prior except the definition of "rational number."]
Conjecture: If x is irrational and $y \neq 0$, then xy is irrational or y is irrational.

*** There are four more problems. Do THREE (omit one)
(10 points each for these three.)

8. Given the triangle inequality and results prior to it, resolve this conjecture:
Conjecture: $|x - y| \geq |x| - |y|$.

9. Prove: If n is prime, it cannot be written as the sum of three consecutive positive integers.
[For example, 11 is prime and is not a sum like 3+4+5.]

[Omit one, your choice, of #9-12]

10. Let $f(x) = 3x + 5$.
Prove: For $\varepsilon > 0$ there exists d such that if $|x - 2| < d$, then $|f(x) - 11| < \varepsilon$.

11. [Inequalities from Section 3.1.] Prove this using only Definition 3 ("$a < b$ iff $b - a > 0$") and prior results which include the axioms, but use nothing after Definition 3:
Theorem: If $a < b$ and $c < d$, then $a - d < b - c$.