Proof, Chapter 4, sets. Name ______________________________
Fall 2014. initial each sheet you use

1. (5 pts) Define *least upper bound* (= supremum) in sentence-form.

2. (3 pts) What is our **name** for the logical equivalence that is the equivalent in logic of this set-theory fact? $S \cup T \subset R$ iff $S \subset R$ and $T \subset R$.

3. (24 pts) True or false? [If true, just say so. However, if it is false, give a counterexample.]
a) T F 3 is an upper bound of the empty set.

b) T F If $f(x) < c$ for all x in the domain of f, then $\sup\{f(x)\} < c$.

c) T F If $S \cap T \neq \varnothing$, then $\sup(S \cap T) \leq \sup(S) + \sup(T)$.

d) T F $S \cap T \in \boldsymbol{P}(S) \cap \boldsymbol{P}(T)$. [$\boldsymbol{P}$ denotes the power set.]

e) T F $\sup\{f(x) + g(x)\} \leq \sup\{f(x)\} + \sup\{g(x)\}$.

Read These Instructions! If it was done in the text, in class, or on the homework, **do it again here**. **Make steps with simple one-step justifications**. **Justify each step**.

To prove something, do not use something very similar or more sophisticated. You are responsible for knowing what is prior. For the next proofs, true assertions without immediate connections to prior results will not receive full credit. True statements that rely on two or more reasons at once will not receive full credit.

4. (10 pts) Prove: If $S \subset T$ and T is bounded, then S is bounded.

[Do these on separate paper provided.]

5. (10 pts) Conjecture: If, for all $\varepsilon > 0$, $d \leq c + \varepsilon$, then $d \leq c$.

6. (12 pts) Prove: If $\boldsymbol{P}(S) \subset \boldsymbol{P}(T)$, then $S \subset T$

7. (12 pts) Prove: If $\sup\{g(x)\}$ is finite and $f(x) \leq g(x)$ for all x, then $\sup\{f(x)\} \leq \sup\{g(x)\}$.

8. (12 pts) Prove: If $\sup(S)$ is finite and $\varepsilon > 0$, there exists x in S such that $x > \sup(S) - \varepsilon$.

9. (12 pts) Prove: (c, ∞) does not have a least element.