Final Exam on *Proof*.
Sections 1.1-5.3. Fall 2014

Name:__________________
initial each sheet of extra paper

1. (10 pts) Give the sentence-form definition of
a) $f(S)$

b) even number

2. (6 pts) Are the two sentences equivalent? Yes or No.
a) Let $f(x) = x^2$ Let $f(a) = a^2$

b) $x^2 + x = x(x + 1)$ $a^2 + a = a(a + 1)$

c) $S = \{x \mid x^2 < 7\}$ $S = \{n \mid n^2 < 7\}$

3. (9 pts) What makes a sentence open? Give a good example. What other types of sentences with one variable are there? Give a good example of each.

4. (8 pts) These might have grammatical mistakes or unconventional usages. Which ones? Give a short and specific indication of the error. $\boldsymbol{P}$ denotes the power set.

a) $S \in \boldsymbol{P}(T)$

b) $S \cap T \Rightarrow x \in S$ and $x \in T$.

c) $\{3, 7\} \subset \{1, 2, 3, ..., \infty\}$

d) Let $x \in f(S)$.

5. (10 pts) Define # by $x\#y = 3x - 2y$.
Solve for x: $5\#x = (4\#5)((2x)\#2)$

6. (24 pts) True or False? No reason required.

a) T F If S is not bounded and T is not bounded, then $S \cap T$ is not bounded.

b) T F $||x| - |c|| \leq |x - c|$.

c) T F If $a^2 < b^2$ and $a < b$, then $a^3 < b^3$.

d) T F If $x > 0$ and $1/x < 1/c$, then $x > c$.

e) T F If C => not D, then D => not C

f) T F For each $x \in (3, 5]$ there exists $y \in (3, 5]$ such that $y > x$.

7. (16 pts) Suppose this is given: If $x < 4$, then $f(x) \geq 8$.
Determine which of these follow logically.

a) FL not FL If $f(x) < 6$, then $x > 4$.

b) FL not FL If $|f(x)| < 7$, then $|x| > 3$.

c) FL not FL If $|x| \leq 2$, then $f(x) \geq 9$.

d) FL not FL $x < 4$ or $f(x) < 8$.

8. (15 pts) Give the negation (in positive form) of these. [You may need to translate first.]
a) [f and S are given] If $x \in S$, then $f(x) > 7$.

b) [f, a, and L are given.] For $\varepsilon > 0$ there exists $\delta > 0$ such that $|f(x) - L| < \varepsilon$ if $0 < |x - a| < \delta$.

c) [About piles of chips] There is a pile with fewer than 5 chips.

9. (6 pts) State the logical equivalences we know by these names:
a) A Hypothesis in the Conclusion

b) A Version of the Contrapositive.

10. (12 pts) Make a truth table, with **all** relevant columns, to determine if "(not *A*) or (not *B*)" is logically equivalent to "*A* => not *B*." [As always, put the column for *A* on the left.] Be sure to conclude if they are or are not logically equivalent.

^^^^^^^^^^^^^^^ **Proofs and Resolutions**

Instructions:

- Do proofs from the **definitions** (not later theorems)
- **Do not cite very similar results** to "prove" things.
- **Cite reasons** for each step.
- Counterexamples must be complete.
- For conjectures, state if they are true or false and prove them if true and disprove them if false.

(12 pts each. Seven problems total 84 points) Use the blank paper provided. Initial each sheet you use.

Do the next four:

11. Prove: If x is rational and x/y is irrational, then y is irrational.

12. Prove: For all $n \geq 1$, $3(12^n) + 8$ is divisible by 11.

13. Conjecture: For $\varepsilon > 0$ there exists c such that if $x > c$, then $1/\sqrt{x} < \varepsilon$.

14. Conjecture: If $a \le b + \varepsilon$ for all $\varepsilon > 0$, then $a \le b$.

-------- Do **three** of the remaining five.

15. As usual, $f{:}A \to B$ and $g{:}B \to C$. Prove: If $g \circ f$ is one-to-one and f is onto B, then g is one-to-one.

16. Prove: If $x > 8$ there exists $y > 6$ such that $4y < 3x$.

[SKIP TWO of 15-19. Be clear which ones you want to be scored.]

17. Conjecture: If $g \circ f$ is onto C, then g is onto C.

18. Let f and g have domain (0, 1) and the sups both be finite.
Conjecture: If $f(x) < g(x)$ for all x, then $\sup\{f(x)\} < \sup\{g(x)\}$.

19. Conjecture: If $\boldsymbol{P}(T) \subset \boldsymbol{P}(S)$, then $T \subset S$.