Proof: Thou Shalt Not! by Warren Esty To prove a result

1) Do not Begin your proof with the result. (It is not prior.)

[However, you may note it is logically equivalent to some other formulation, and that you intend to prove that second formulation instead.]

The statement of the result you wish to prove probably has some letters [For example, S or f]. Your proof may use those given letters.

2) Do not Use other, new, letters before you have defined them.

The first time a letter appears in a proof it must be

i) given in the problem, or

ii) used as representative of all of its kind (in a "representative case" proof). These are often introduced with the word *Let*, for example, "Let  $\varepsilon > 0$ .", or

iii) asserted to exist by some prior result (in which case you write "There exists ... by Theorem x.x."), or

iv) newly (and clearly) defined in terms of previously given things (for example, "Choose  $\delta = \epsilon/5$ " after  $\epsilon$  has been given).

New letters that appear in an argument they must be clearly defined or quantified, or else the argument might go wrong at that stage. Be careful with the existence and generalization of new letters.

3) Do not "Let" a letter have a property without justification that it really could have that property.

[When we, for example, "Let x be in S," the letter "x" has not been previously used and is not given in the problem. It is a name for an arbitrary member of S. That type of "Let" is legal. However, if f is given by  $f(x) = x^2$ , we cannot "Let f(x)+2 = 1." We don't know there exists such an x (and there is not). "Letting" a new letter have two properties at once is illegal unless you can cite a reason that there really exists such a thing.]

4) Do not Use the same letter for two different things in the same proof.

5) Do not Use true assertions that are not clearly prior.

(Non-trivial assertions must be from our list of prior results. Truth is not enough. You are not allowed to use assertions just because they are true. Of course, much-lower level results can be used without citing them. If you don't know if some result you want to use is really prior, label it a "claim".)

6) Do not Use a theorem to prove itself.

7) Do not Omit "there exists" when it is meant.

8) Do not Use "=" to connect things that are not really =.

9) Do not Fail to use "=" if you really mean =.

10) Do not Construct cases instead of using general representative cases (when a result is supposed to hold in general).

11) Do not Give counterexamples which fail to give values to **all** the letters.