

1. (12 pts) Give the negation, in positive form, of these.

a) [f is given] If $x > 7$, then $f(x) \leq 15$.

b) [f is given] $f(x) > 1 \Rightarrow x < 5$ or $x \geq 10$.

c) [$\{a_n\}$ is given] For any m and any n^* , there is $n > n^*$ such that $a_n > m$.

d) If $a > 0$, $ax^2 + bx + c = 0$ has no solutions whenever $c > 0$.

e) There is always a solution to $mx + b = c$.

f) All students in this class have a GPA of at least 3.00.

2. (8 pts) The textbook sentence-form definition of “ b is an upper bound of S ” is
“ b is an upper bound of S iff if $x \in S$, then $x \leq b$.”

For each part give a sentence that is equivalent to (and, therefore, could replace)
“ c is an upper bound of T ”
for the following reasons:

a) by definition:

b) As part (a), but with a different quantified letter:

c) by a logical equivalence:

d) using English:

1. (12 pts) Give the negation, in positive form, of these.

a) [f is given] If $x > 7$, then $f(x) \leq 15$.

There exists $x > 7$ such that $f(x) > 15$.

b) [f is given] $f(x) > 1 \Rightarrow x < 5$ or $x \geq 10$.

There exists x such that $f(x) > 1$ and $x \geq 5$ and $x < 10$.

c) [$\{a_n\}$ is given] For any m and any n^* , there is $n > n^*$ such that $a_n > m$.

There exist m and n^* such that for all $n > n^*$, $a_n \leq m$.

d) If $a > 0$, $ax^2 + bx + c = 0$ has no solutions whenever $c > 0$.

There exists a , b , and c , such that $a > 0$, $c > 0$ and $ax^2 + bx + c = 0$ has a solution.

e) There is always a solution to $mx + b = c$.

There exists m , b , and c , such that $mx + b = c$ does not have a solution.
[$mx + b \neq c$ for all x .]

f) All students in this class have a GPA of at least 3.00.

There exists a student in this class with a GPA of less than 3.00.

2. (8 pts) The textbook sentence-form definition of “ b is an upper bound of S ” is
“ b is an upper bound of S iff if $x \in S$, then $x \leq b$.”

For each part give a sentence that is equivalent to (and, therefore, could replace)

“ c is an upper bound of T ”

for the following reasons:

a) by definition: If $x \in T$, then $x \leq c$

b) As part (a), but with a different quantified letter: If $t \in T$, then $t \leq c$

c) by a logical equivalence: If $x > c$, then x is not in T .

d) using English: All the elements of T are less than or equal to c .