Multivariable Calculus M273
Fall 2018
December 12, 2018
Final Exam

Name: ______________________________

Section: ____

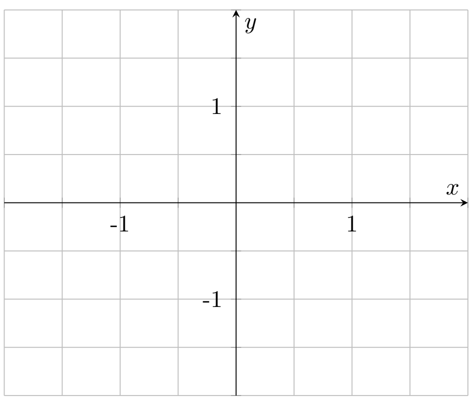
Instructor Name: ______________________

Instructions: Closed book. No calculator allowed. Double-sided exam. NO CELL PHONES.
Show all work and use correct notation to receive full credit! Write legibly.

$\mathrm{d}A = r\,\mathrm{d}r\,\mathrm{d}\theta$	$\mathrm{d}V = r\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$	$\mathrm{d}V = \rho^2\,\sin\phi\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}\theta$

1. Consider the integral $\displaystyle\iint_{\mathcal{D}} \frac{1}{1-y^4}\,dA = \int_0^1 \int_0^{\sqrt[4]{x}} \frac{1}{1-y^4}\,dy\,dx.$

   (a) (1 Credit) Sketch $\mathcal{D}$, the region of integration.



   (b) (2 Credits) Calculate $\displaystyle\int_0^1 \int_0^{\sqrt[4]{x}} \frac{1}{1-y^4}\,dy\,dx$ by switching the order of integration.

2. (1 Credit) True/False.

   (a) ◯ True ◯ False The two lines $\mathcal{L}_1 : \mathbf{r}_1(t) = \langle 10-4t, 3+t, 4+t\rangle$ and $\mathcal{L}_2 : \mathbf{r}_2(t) = \langle 6, mt, 9-t\rangle$ intersect if $m = 1$.

   (b) ◯ True ◯ False The vectors $\mathbf{v}_1 = \langle 1, \lambda, -2\rangle$ and $\mathbf{v}_2 = \langle 4\lambda, 3, 7\rangle$ are orthogonal when $\lambda = 2$.

   (c) ◯ True ◯ False Consider the sphere $x^2+y^2+z^2 = 9$. The plane $x = 3$ is the tangent plane to the sphere at the point $(3, 0, 0)$.

   (d) ◯ True ◯ False $\int_1^2\int_3^4 x^2 e^y \, \mathrm{d}y \, \mathrm{d}x = \left(\int_1^2 x^2 \, \mathrm{d}x\right)\left(\int_3^4 e^y \, \mathrm{d}y\right)$

3. (1 Credit) The following equations are all results from theorems we learned this semester.

$$\oint_{\partial D} \mathbf{F}\cdot \mathrm{d}\mathbf{r} = \iint_D \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) \mathrm{d}A \qquad \text{I}$$

$$\int_a^b\int_c^d f \ \mathrm{d}y\mathrm{d}x = \int_c^d\int_a^b f \, \mathrm{d}x\mathrm{d}y \qquad \text{II}$$

$$\oint_{\partial \mathcal{S}} \mathbf{F}\cdot \mathrm{d}\mathbf{r} = \iint_{\mathcal{S}} \mathrm{curl}(\mathbf{F})\cdot \mathrm{d}\mathbf{S} \qquad \text{III}$$

$$\iint_{\partial \mathcal{W}} \mathbf{F}\cdot \mathrm{d}\mathbf{S} = \iiint_{\mathcal{W}} \mathrm{div}(\mathbf{F}) \, \mathrm{d}V \qquad \text{IV}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} \qquad \text{V}$$

$$\int_C \nabla f \cdot \mathrm{d}\mathbf{r} = f(Q) - f(P) \qquad \text{VI}$$

Match each of the following theorems with its corresponding equation above.

   (a) Divergence Theorem
   ◯ I ◯ II ◯ III ◯ IV ◯ V ◯ VI

   (b) Green's Theorem
   ◯ I ◯ II ◯ III ◯ IV ◯ V ◯ VI

   (c) Fundamental Theorem for Conservative Vector Fields
   ◯ I ◯ II ◯ III ◯ IV ◯ V ◯ VI

   (d) Stokes' Theorem
   ◯ I ◯ II ◯ III ◯ IV ◯ V ◯ VI

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4. A radioactive substance with strength

$$P(x, y, z) = e^{-x^2-y^2-(z+100)^2}$$

is suddenly discharged. A person standing at the point $(1, 1, -100)$ must move away, in the direction of maximum <u>decrease</u> of radiation. Note that the gradient of P is given by

$$\nabla P(x, y, z) = \langle -2xe^{-x^2-y^2-(z+100)^2}, -2ye^{-x^2-y^2-(z+100)^2}, -2(z + 100)e^{-x^2-y^2-(z+100)^2} \rangle.$$

(a) (1 Credit) A person standing at the point $(1, 1, -100)$ must move away, in the direction of maximum <u>decrease</u> of radiation. What direction should he/she choose to move?

○ $\langle 1, 1, 0\rangle$ ○ $\langle 2, 3, 0\rangle$ ○ $\langle -1, -1, 0\rangle$ ○ $\langle -2, -3, 0\rangle$ ○ $\langle 1, -1\rangle$

(b) (1 Credit) A person standing at the point $(1, 1, -100)$ must move away, in the direction of maximum <u>decrease</u> of radiation. What is the maximum rate of <u>decrease</u> in radiation?

○ $-\sqrt{2}$ ○ $-e^{-3}\sqrt{13}$ ○ $-2\sqrt{2}e^{-2}$ ○ $\sqrt{2}$ ○ 0

(c) (1 Credit) The person standing at the point $(1, 1, -100)$ decided to move in the direction $\langle 0, 1, 0\rangle$. What is the rate of change in radiation in this direction?

○ $2e^3$ ○ $-2e^{-3}$ ○ 0 ○ $-2e^{-2}$

5. (2 Credits) Find the equation of the plane containing the points $P = (1, 2, 1)$, $Q = (3, 2, -1)$, $R = (1, 1, 1)$.

6. (2 Credits) Find the tangent line to the curve parametrized by

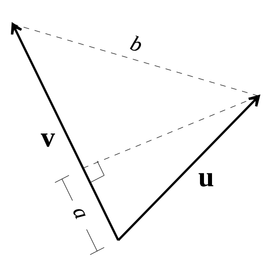
$$\mathbf{r}(t) = \left\langle t + \cos(t), te^t, \ln(1+t) \right\rangle \quad , \quad t > -1$$

at the point where $t = 0$.

7. (2 Credits) Using the Divergence Theorem, find the flux of the vector field $\mathbf{F}(x, y, z) = \langle 3x, 4y, -2z \rangle$ outwards across the surface of the box $\mathcal{W} = [0,1] \times [0,2] \times [-1,1]$.

$$\iint_{\partial \mathcal{W}} \mathbf{F} \cdot \mathrm{d}\mathbf{S} =$$

8. (2 Credits) Given $\mathbf{u} = 5\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $\mathbf{v} = 4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$, find the **lengths** a and b pictured below.



$a =$

$b =$

9. (2 Credits) Let $\mathcal{S}$ be the cone given by $\{(x, y, z) \in \mathbb{R}^3 \mid z = \sqrt{x^2 + y^2} \text{ and } 0 \leq z \leq 1\}$ and let the vector field $\mathbf{F}$ be given by $\mathbf{F}(x, y, z) = \left\langle -yz, xz, z^2 \right\rangle$.

   Given that $\partial\mathcal{S}$ is parametrized by $\mathbf{r}(t) = \langle \cos(t), \sin(t), 1 \rangle$, $0 \leq t \leq 2\pi$, use Stokes' Theorem to find the flux of curl($\mathbf{F}$) upwards across $\mathcal{S}$, that is, find $\iint\limits_{\mathcal{S}} \operatorname{curl}(\mathbf{F}) \cdot \mathrm{d}\mathbf{S}$.

$\iint\limits_{\mathcal{S}} \operatorname{curl}(\mathbf{F}) \cdot \mathrm{d}\mathbf{S} =$

10. (2 Credits) Consider the surface $\mathcal{S}$ parametrized by $\mathbf{r}(u, v) = \langle u + v, u - v, 2u + 3v \rangle$ where $0 \leq u \leq 1$ and $0 \leq v \leq 1$. Find the surface area of $\mathcal{S}$.

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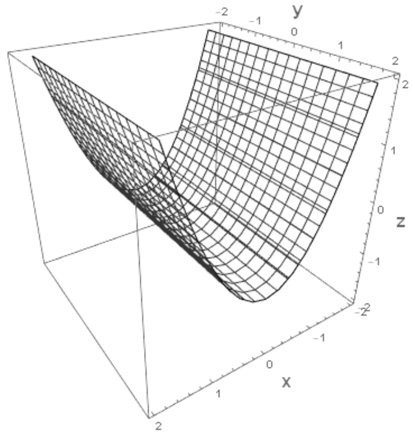
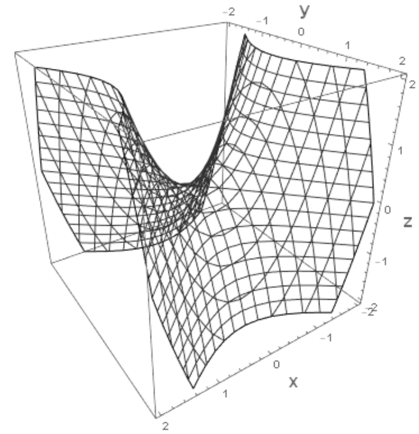
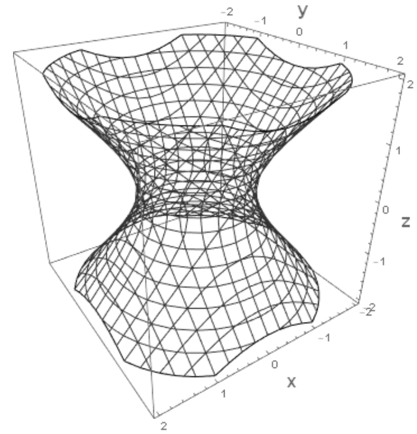
11. (1 Credit) Consider the following quadratic surfaces in $\mathbb{R}^3$.



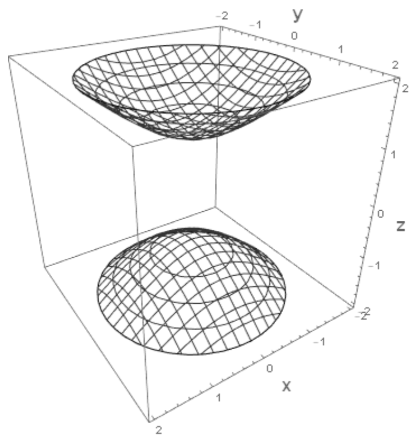
I



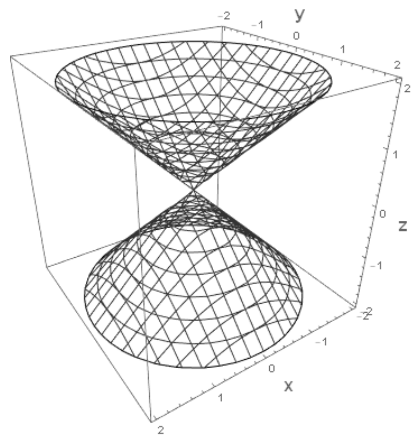
II



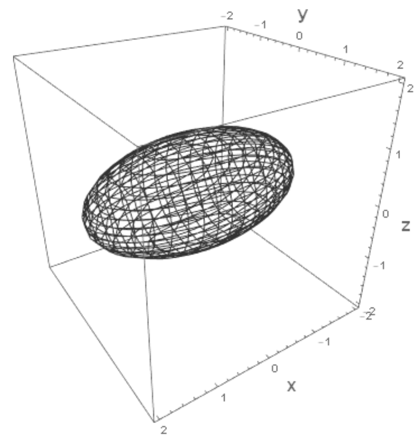
III



IV



V



VI

Match each of the following equations with its corresponding graph above.

(a) $z = x^2 - y^2$
◯ I ◯ II ◯ III ◯ IV ◯ V ◯ VI

(b) $x^2 + y^2 = z^2 - 1$
◯ I ◯ II ◯ III ◯ IV ◯ V ◯ VI

(c) $z^2 = x^2 + y^2$
◯ I ◯ II ◯ III ◯ IV ◯ V ◯ VI

(d) $z = x^2 - 1$
◯ I ◯ II ◯ III ◯ IV ◯ V ◯ VI

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12. (2 Credits) Let $\mathcal{E}$ be the region in $\mathbb{R}^3$ where $1 \leq x^2 + y^2 + z^2 \leq 4$ and $x, y, z \geq 0$ (i.e., in the first octant). Evaluate the integral

$$\iiint_{\mathcal{E}} (x^2 + y^2 + z^2)^{3/2} \, \mathrm{d}V.$$

Problem	1	2	3	4	5	6	7	8	9	10	11	12	Total
Credit	3	1	1	3	2	2	2	2	2	2	1	2	23
Credit Points													

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