**M273Q Multivariable Calculus**
**An Old Exam 2**

Name and section: ______________________________

Instructor's name: ______________________________

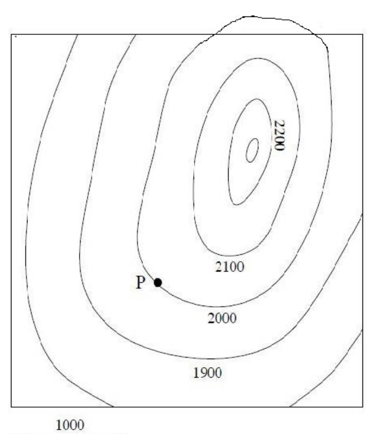
<u>Instructions</u>: Closed book. No calculator allowed. Double-sided exam. NO CELL PHONES. **Show all work and use correct notation to receive full credit!** Write legibly.

1. (2 credit ____) Let $f(x,y,z) = \sin(xyz) - x - 2y - 3z$. Note that

$$\nabla f(x,y,z) = \langle -1 + yz\cos(xyz),\ -2 + xz\cos(xyz),\ -3 + xy\cos(xyz)\rangle\,.$$

Find an equation for the tangent plane to the surface $\sin(xyz) = x + 2y + 3z$ at the point $(2,-1,0)$.

2. On the topographical map below, the level curves for the height function h(x, y) are marked (in meters); adjacent level curves represent a difference of 100 meters in height. A scale is given.




(a) (1 credit ____) At the point $P$, sketch a vector pointing in the direction of the gradient of $h$.

(b) (1 credit ____) Mark on the map a point $Q$ at which $h = 2000$, $\dfrac{\partial h}{\partial x} = 0$ and $\dfrac{\partial h}{\partial y} < 0$.

| Question: | 1 | 2 | Total |
|---|---|---|---|
| Credit | 2 | 2 | 4 |
| GPA Credit Points Earned | | | |

3. (2 credit ____) Let

$$w(x,y,z) = xy + yz + zx, \quad x(r,\theta) = r\cos\theta, \quad y(r,\theta) = r\sin\theta, \quad z(r,\theta) = r\theta.$$

Find $\dfrac{\partial w}{\partial r}$, where $r = 2, \theta = \pi/2$.

| Question: | 3 | Total |
|---|---|---|
| Credit | 2 | 2 |
| GPA Credit Points Earned | | |

4. Evaluate the limit or show that the limit does not exist.

(a) (1 credit ____) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2}$

(b) (1 credit ____) $\displaystyle\lim_{(x,y)\to(1,1)} \frac{4+x-y}{3+x-3y}$

5. (2 credit ____) Given that $x^3z - 3xy^2 - (yz)^3 = -3$ find $\dfrac{\partial z}{\partial x}$.

| Question: | 4 | 5 | Total |
|---|---|---|---|
| Credit | 2 | 2 | 4 |
| GPA Credit Points Earned | | | |

6. (3 credit ____) Find all critical points of $f(x, y) = x^2 + \frac{1}{3}y^3 - 2xy - 3y$ and classify them (local maximum, local minimum, or saddle) using the Second Derivative Test.

| Question: | 6 | Total |
|---|---|---|
| Credit | 3 | 3 |
| GPA Credit Points Earned | | |

7. (3 credit ____) Find the coordinates of the points on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$ at which the function $f(x, y) = xy$ is maximized and those at which $f$ is minimized.

| Question: | 9 | 10 | Total |
|---|---|---|---|
| Credit | 1 | 2 | 3 |
| GPA Credit Points Earned | | | |

8. Your house lies on the surface $z = f(x, y) = 2x^2 - y^2$ directly above the point $(4, 3)$ in the $xy-$plane.

(a) (1 credit ____) How high above the $xy-$plane do you live?

(b) (1 credit ____) Calculate the gradient of $f$ at the point $(4, 3)$.

(c) (1 credit ____) What is the slope of your lawn as you look from your house directly toward the $z-$axis (that is, along the vector $< -4, -3 >$)?

(d) (1 credit ____) When you wash your car in the driveway, on this surface above the point $(4, 3)$, which way does the water run off? (Give your answer as a two-dimensional vector.)

| Question: | 8 | Total |
| --- | --- | --- |
| Credit | 4 | 4 |
| GPA Credit Points Earned | | |

9. (1 credit ____) At what point on the surface $z = 1 + x^2 + y^2$ is its tangent plane parallel to the plane $z = 5 + 6x - 10y$?

10. Let $f(x, y) = x^7(1 + 2\sin y)$. Note that $f(1, 0) = 1,\ f_x(1, 0) = 7,$ and $f_y(1, 0) = 2$.

    (a) (1 credit ____) Find an equation of the tangent plane to $f$ at $(1, 0)$.

    (b) (1 credit ____) Approximate $(0.9)^7(1 + 2sin(0.2))$.

| Question | Points | Score |
|---|---|---|
| 9 | 1 | |
| 10 | 2 | |
| Total: | 3 | |

| Page: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Total |
|---|---|---|---|---|---|---|---|---|
| Credit | 4 | 2 | 4 | 3 | 3 | 4 | 3 | 23 |
| GPA Credit Points Earned | | | | | | | | |