M273Q Multivariable Calculus **Old Exam 1**

Name and section: _____

Instructor's name: _____

Instructions: Closed book. No calculator allowed. Double-sided exam. NO CELL PHONES.
Show all work and use correct notation to receive full credit! Write legibly.

$$\kappa(s) = \left|\left|\frac{d\mathbf{T}}{ds}\right|\right| \qquad \kappa(x) = \frac{|f''(x)|}{[1+(f'(x))^2]^{3/2}}$$

$$\kappa(t) = \frac{||\mathbf{T}'(t)||}{||\mathbf{r}'(t)||} \qquad \kappa(t) = \frac{||\mathbf{r}'(t) \times \mathbf{r}''(t)||}{||\mathbf{r}'(t)||^3}$$

1. (1 credit ____) Decide whether the following quantities are vectors, scalars, or are nonsensical (that is, the statement is not defined or does not make sense)

   ◯ Vector ◯ Scalar ◯ Nonsense $(\mathbf{u} \cdot \mathbf{v}) \times \mathbf{w}$

   ◯ Vector ◯ Scalar ◯ Nonsense $\|\mathbf{u} \times \mathbf{v}\|$

   ◯ Vector ◯ Scalar ◯ Nonsense $(\mathbf{u} \cdot \mathbf{v})\mathbf{w}$

   ◯ Vector ◯ Scalar ◯ Nonsense $\kappa(t) = \dfrac{||\mathbf{r}'(t) \times \mathbf{r}''(t)||}{||\mathbf{r}'(t)||^3}$

2. (1 credit ____) Determine whether the following equations describe a plane, a line, or neither in $\mathbf{R}^3$:

   ◯ Plane ◯ Line ◯ Neither $\mathbf{r}(t) =< 1, -1, 5 > +t < 0, 2, 3 >$

   ◯ Plane ◯ Line ◯ Neither $x + y + z = 1$

   ◯ Plane ◯ Line ◯ Neither $y = 5 + z$

   ◯ Plane ◯ Line ◯ Neither $x(t) = 7 + 2t,\ y(t) = 11 - 5t,\ z(t) = \dfrac{t}{\pi}$

3. (1 credit ____) Find a **unit** vector parallel to the line $\mathbf{r}(t) =< t + 4, -2 + 2t, -5 - 2t >$.

Problem	1	2	3	Total
Credit	1	1	1	3
GPA Credit Points Earned				

4. Let $\mathbf{a} =< 1, 2, -1 >$ and $\mathbf{b} =< 2, -1, 3 >$.

(a) (1 credit ____) Find $\|\mathbf{a}\|$.

(b) (1 credit ____) Find $\mathbf{b} \times \mathbf{a}$.

(c) (1 credit ____) Find $\mathbf{a} \cdot \mathbf{b}$.

(d) (1 credit ____) Find $3\mathbf{a} - 2\mathbf{b}$.

(e) (1 credit ____) Is the angle between $\mathbf{a}$ and $\mathbf{b}$ acute (less that $\pi/2$), obtuse (greater than $\pi/2$), or neither?

Problem	4	Total
Credit	5	5
GPA Credit Points Earned		

5. (1 credit ____) Find a vector of length 4 that is orthogonal to both $\mathbf{a} =< 1, 2, -1 >$ and $\mathbf{b} =< 2, -1, 3 >$.

6. This question has two parts.

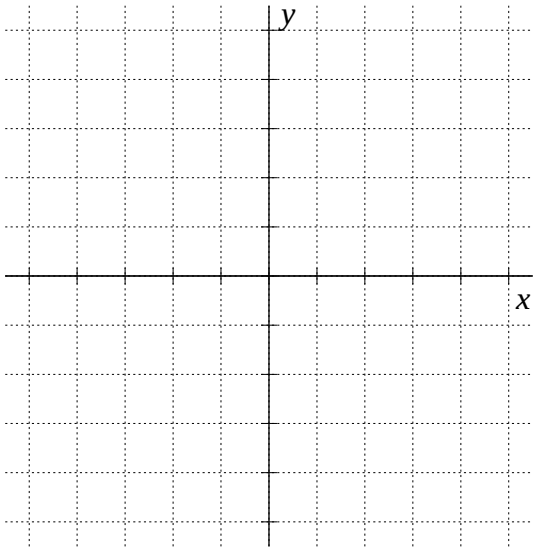
   (a) (1 credit ____) Find the equation of the line through $P(3, 1, 0)$ and $Q(1, 4, -3)$.

   (b) (1 credit ____) Show that the line you found in part (a) is orthogonal to $x(t) = 3t$, $y(t) = 3+8t$, $z(t) = -7 + 6t$.

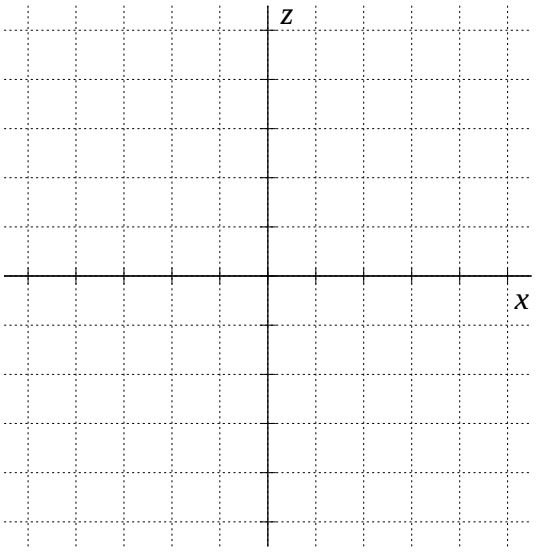
7. (2 credit ____) Find an equation for the tangent line to the curve $\mathbf{r}(t) =< t, t^2, t^3 >$ at the point $(1, 1, 1)$.

Problem	5	6	7	Total
Credit	1	2	2	5
GPA Credit Points Earned				

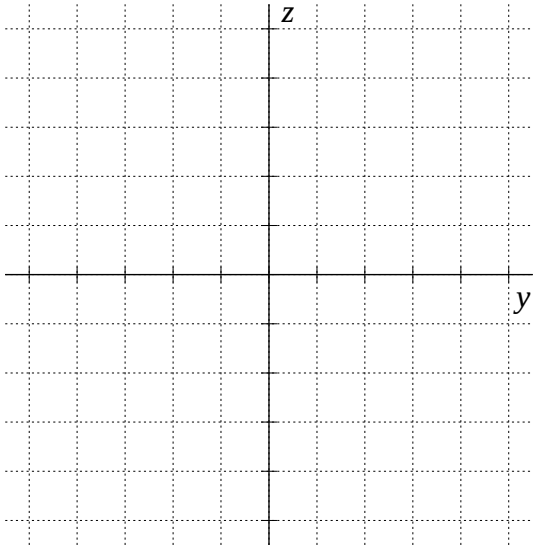
8. (4 credit ____) Provide a clear sketch of the following traces for the quadratic surface $z - x^2 - y^2 = 0$ in the given planes.



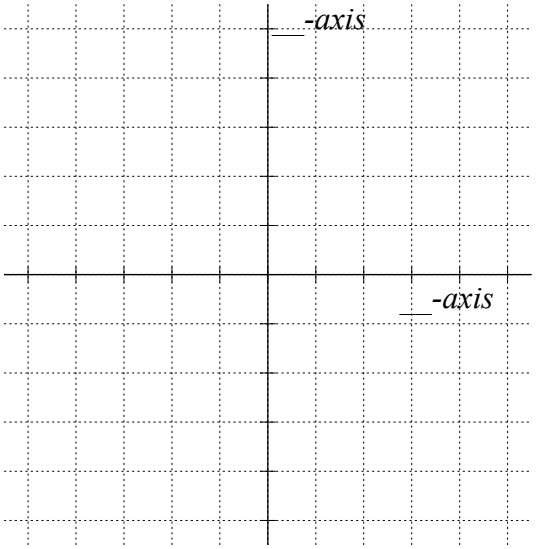
$xy-$plane



$xz-$plane

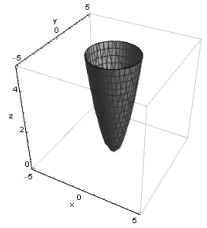


$yz-$plane

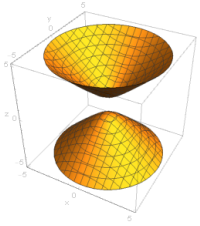


$z = 1$ label the appropriate axes.

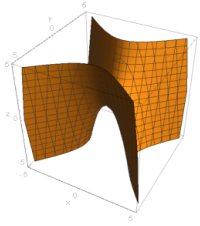
9. (1 credit ____) Based on the traces you found above, identify the graph of $z - x^2 - y^2 = 0$ by circling the figure number.



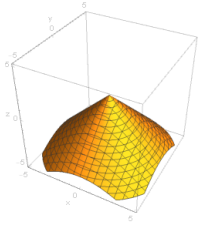
I.



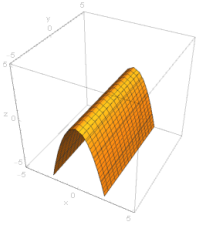
II.



III.



IV.



V.

Problem	8	9	Total
Credit	4	1	5
GPA Credit Points Earned			

10. Given position $\mathbf{r}(t) = \langle 6\sin t, 6\cos t, 8t\rangle$, $a > 0$ at time t, find the following:

(a) (1 credit ____) The unit tangent vector $\mathbf{T}(t) =$

(b) (1 credit ____) The unit normal vector $\mathbf{N}(t) =$

(c) (1 credit ____) The curvature of the graph of $\mathbf{r}(t)$ at $t = 0$, $\kappa(0) =$ _____.

Problem	10	Total
Credit	3	3
GPA Credit Points Earned		

11. Let $\mathbf{c}(t) =< 6\sin 2t, 6\cos 2t >$.

(a) (1 credit ____) Sketch $\mathbf{c}(t)$ for $0 \leq t \leq \pi$.

(b) (1 credit ____) Label the point corresponding to $\mathbf{c}\left(\frac{\pi}{4}\right)$ on your graph.

(c) (1 credit ____) Calculate $\mathbf{c}'\left(\frac{\pi}{4}\right)$.

(d) (1 credit ____) Sketch the vector $\mathbf{c}'\left(\frac{\pi}{4}\right)$ at the appropriate point on your graph.

12. Given $\mathbf{a} =< 3, -4, 4 >$ and $\mathbf{b} =< 2, 2, 1 >$, find vectors $\mathbf{a}_{||\mathbf{b}}$ and $\mathbf{a}_{\perp\mathbf{b}}$:

(a) (1 credit ____) $\mathbf{a}_{||\mathbf{b}} =$ _____

(b) (1 credit ____) $\mathbf{a}_{\perp\mathbf{b}} =$ _____

(c) (1 credit ____) Show that $\mathbf{a}_{\perp\mathbf{b}}$ is orthogonal to $\mathbf{b}$.

Problem	11	12	Total
Credit	4	3	7
GPA Credit Points Earned			

13. A curve is parameterized by $\mathbf{r}(t) =< 3 + \cos 3t, 3 - \sin 3t, 4t >$.

(a) (1 credit ____) Find the arc length of the piece of the curve $0 \leq t \leq \dfrac{2\pi}{3}$.

(b) (1 credit ____) Re-parameterize the curve with respect to arc length measured from the point where $t = 0$ in the direction of increasing t.

Problem	13	Total
Credit	2	2
GPA Credit Points Earned		

14. (2 credit ___) Find an equation for the plane that contains the points $A(1,2,3)$, $B(2,4,2)$ that is parallel to $\mathbf{v} =< -3,-1,-2 >$.

Question	Points	Score
14	2	
Total:	2	

Page:	1	2	3	4	5	6	7	8	Total
Credit	3	5	5	5	3	7	2	2	32
GPA Credit Points Earned									

Name and section: _____