M273Q, Old Final A Name:______________________________

Instructions: Closed book. No calculator allowed. Show all work and use correct notation to receive full credit! Write legibly.

1. Let $\mathbf{F}$ be a vector field in $\mathbb{R}^3$ and f a scalar function of three variables. For each of the following, state whether the operations shown produce a vector field, a scalar function, or whether they cannot be computed, in which case the statement is nonsense.

   (a) **Vector** **Scalar** **Nonsense** $\nabla \cdot \mathbf{F}$

   (b) **Vector** **Scalar** **Nonsense** $\nabla(\nabla \cdot \mathbf{F})$

   (c) **Vector** **Scalar** **Nonsense** $\nabla \times (\nabla \times \mathbf{F})$

   (d) **Vector** **Scalar** **Nonsense** $(\nabla f) \times \mathbf{F}$

   (e) **Vector** **Scalar** **Nonsense** $(\nabla \cdot \mathbf{F}) \cdot \mathbf{F}$

2. Given the vectors $\mathbf{a} =< 2, 1, 0 >, \mathbf{b} =< 2, -1, 2 >$, and $\mathbf{c} =< 0, 2, 1 >$, find:

   (a) A vector of length 7 that is perpendicular to both $\mathbf{a}$ and $\mathbf{b}$.

   (b) An equation for the plane that is parallel to both $\mathbf{a}$ and $\mathbf{b}$ and that goes through the point $(-1, 1, 2)$.

   (c) The volume of the parallelopiped spanned by $\mathbf{a}, \mathbf{b}$, and $\mathbf{c}$.

   (d) The cosine of the angle between $\mathbf{a}$ and $\mathbf{c}$.

3. Let $\mathbf{r}(t) =< t^2, -2t, t >$ and $f(x, y, z) = x^2(y + z)$.

   (a) At the point $(1, -2, 1)$ in what direction does f increase most rapidly?

   (b) Find the rate of change of f in the direction tangent to the curve $\mathbf{r}(t)$ at the point $(1, -2, 1)$.

   (c) Use the chain rule to calculate $\frac{d}{dt}(f(\mathbf{r}(t))$ at $t = 1$.

4. Given that $\mathbf{r}(t) =< \cos 3t, \sin 3t, 4t >$ is position at time t, find:

   (a) the velocity at time $t = \pi$.

   (b) the speed at time $t = \pi$.

   (c) the acceleration at time $t = \pi$.

   (d) the length of the path of motion at time $t = \pi$.

   (e) an equation for the tangent line to $\mathbf{r}(t)$ at the point $(-1, 0, 4\pi)$.

5. Compute $\iint_{\mathcal{D}} x + y \, dA$ where $\mathcal{D}$ is the triangular domain with vertices $(-1, 1), (1, 1), (0, 0)$.

6. Set up but <u>do not evaluate</u>, iterated triple integrals with the appropriate limits for $\iiint_W x^2 + y^2 \, dV$ where $\mathcal{W}$ is the solid lying inside $x^2 + y^2 + z^2 = 2$ and above $z = 1$, in:

   (a) rectangular coordinates

   (b) cylindrical coordinates

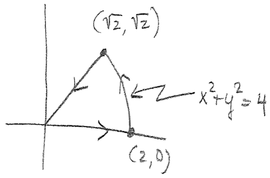
   (c) spherical coordinates

7. Compute $\int_{\mathcal{C}} xz \, ds$, $\mathcal{C}$ is the straight line segment from $(1, 2, 3)$ to $(3, 1, 1)$.

8. (a) Is $\mathbf{F}(x, y, z) =< 2x \cos y, \cos y - x^2 \sin y, z >$ conservative? (justify).

   (b) Calculate $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$, where $\mathcal{C}$ is the curve $\mathbf{r}(t) =< te^t, t\pi, (1 + t)^2 >$, $0 \le t \le 1$.

9. Use Green's Theorem to calculate $\int_{\mathcal{C}} (x - y^3)dx + (x^3 - y)dy$, where $\mathcal{C}$ is the closed curve bounding the wedge shaped region pictured.



10. Find the surface area of the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the plane $z = 0$.

11. Let E be the solid region that is bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by $z = \sqrt{9 - x^2 - y^2}$. Calculate the flux of $\mathbf{F}(x, y, z) =< xy^2, yx^2, \frac{1}{3}z^3 >$ outwards across the boundary surface of $\mathcal{E}$.