M273Q, Old Final B Name:______________________________

Instructions: Closed book. No calculator allowed. Show all work and use correct notation to receive full credit! Write legibly.

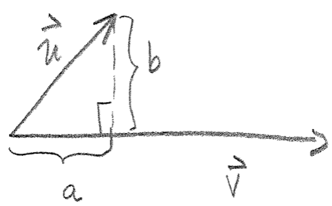
1. True or False? Circle ONE answer for each.

   True or False: If $\mathbf{F}$ is a vector field, then div $\mathbf{F}$ is a vector field.

   True or False: If $\mathbf{F}$ and $\mathbf{G}$ are vector fields, then curl $(\mathbf{F} \cdot \mathbf{G}) =$ curl $\mathbf{F} \cdot$ curl $\mathbf{G}$.

   True or False: If $\mathcal{S}$ is a sphere and $\mathbf{F}$ is a constant vector field, then $\iint_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S} = 0$.

2. Given the vectors $\mathbf{v} =< 2, 1, -2 >, \mathbf{u} =< 3, 2, 1 >$, find the lengths of a and b pictured below:



3. (a) Find an equation for the line through the points $(1, 2, 3)$ and $(-1, 5, 4)$.
   (b) Find an equation for the plane that is perpendicular to the line in part 3a and passes through the point $(4, 0, 1)$.
   (c) At what point do the line in part 3a and the plane in part 3b intersect?

4. Given $f(x, y, z) = \dfrac{x}{1 + xyz}$:
   (a) At the point $(1, 0, 2)$ in what direction does f increase most rapidly?
   (b) At the point $(1, 0, 2)$ what is the rate of change of f in the direction of $< 3, -4, 0 >$?

5. Given $x = r\cos\theta$ and that r and θ depend on t in such a way that when $t = 0 : r = 2, \theta = \pi/4, \dfrac{dr}{dt} = 3, \dfrac{d\theta}{dt} = \pi$, find $\dfrac{dx}{dt}$ at $t = 0$.

6. Calculate $\displaystyle\int_0^1 \int_{\sqrt{x}}^1 \sqrt{1 + y^3}\, dy\, dx$.

7. Let $\mathcal{W}$ be the region above the cone $z = \sqrt{x^2 + y^2}$ and below the plane $z = 1$. Calculate $\displaystyle\iiint_{\mathcal{W}} x^2 + y^2\, dV$.

8. Verify that the vector field $\mathbf{F}(x, y, z) =< 2xy + z, x^2 + 1, x + 2z >$ is conservative and calculate the work done by $\mathbf{F}$ in moving an object from $(2, -1, 1)$ to $(1, 1, 0)$.

9. Calculate $\oint_{\mathcal{C}} -y^2\, dx + xy\, dy$ where $\mathcal{C}$ is the counterclockwise oriented simple closed curve consisting of the piece of the parabola $y = 1 - x^2$ between $(-1, 0)$ and $(1, 0)$ together with the piece of the $x-$axis between $(-1, 0)$ and $(1, 0)$.

10. Find the surface area of the part of surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$.

11. Let $\mathcal{S}$ be the part of the paraboloid $z = 4 - x^2 - y^2$ with $z \geq 0$, oriented with upwards pointing normal vector, and let $\mathbf{F}(x, y, z) =< -y, x, z >$. Using Stokes' Theorem, calculate $\iint_{\mathcal{S}} (curl(\mathbf{F})) \cdot d\mathbf{S}$.

12. Let $\mathbf{F}(x, y, z) =< y, x, z^2 >$ and let $\mathcal{S}$ be the closed surface consisting of the cone $z = \sqrt{x^2 + y^2}$, $0 \leq z \leq \sqrt{2}$, and the spherical cap $z = \sqrt{4 - x^2 - y^2}$, $\sqrt{2} \leq z \leq 2$. Using the divergence theorem, calculate the flux, $\iint_{\mathcal{S}} \mathbf{F} \cdot d\mathcal{S}$, of $\mathbf{F}$ outwards across $\mathcal{S}$.